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A set-covering based heuristic algorithm for the periodic vehicle routing problem

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ABSTRACT

We present a hybrid optimization algorithm for mixed-integer linear programming, embedding both heuristic and exact components. In order to validate it we use the periodic vehicle routing problem (PVRP) as a case study. This problem consists of determining a set of minimum cost routes for each day of a given planning horizon, with the constraints that each customer must be visited a required number of times (chosen among a set of valid day combinations), must receive every time the required quantity of product, and that the number of routes per day (each respecting the capacity of the vehicle) does not exceed the total number of available vehicles. This is a generalization of the well-known vehicle routing problem (VRP). Our algorithm is based on the linear programming (LP) relaxation of a set-covering-like integer linear programming formulation of the problem, with additional constraints. The LP-relaxation is solved by column generation, where columns are generated heuristically by an iterated local search algorithm. The whole solution method takes advantage of the LP-solution and applies techniques of fixing and releasing of the columns as a local search, making use of a tabu list to avoid cycling. We show the results of the proposed algorithm on benchmark instances from the literature and compare them to the state-of-the-art algorithms, showing the effectiveness of our approach in producing good quality solutions. In addition, we report the results on realistic instances of the PVRP introduced in Pacheco et al. (2011) [24] and on benchmark instances of the periodic traveling salesman problem (PTSP), showing the efficacy of the proposed algorithm on these as well. Finally, we report the new best known solutions found for all the tested problems.

1. Introduction

Routing problems have been widely studied due to the economic importance of developing efficient techniques for optimization in transportation. Many real-world applications in transportation systems require finding a set of minimum cost routes to be performed by a fleet of vehicles, that satisfy orders of products requested by customers, over a given planning horizon (e.g. one week). In some applications, it can be useful to visit customers a different number of times, depending on the amount of products requested. In addition, certain days of the planning horizon can be specified by the

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customers as available for the delivery of the product. Taking into account several days of planning for routing problems is an interesting variant of the classical vehicle routing problem (VRP), known as the periodic vehicle routing problem (PVRP). This problem is very important in real-world applications such as distribution for bakery companies [24], or blood product distribution [21].

In this work we propose a hybrid optimization heuristic algorithm that can be applied to mixed integer linear programming problems. It is based on the solution of the linear programming (LP) relaxation of an integer linear programming formulation of the problem. The LP-relaxation can be solved by a general purpose solver or by column generation techniques, with exact or heuristic methods. The LP-solution can be then used to guide the heuristic algorithm for obtaining an integer solution.

This heuristic algorithm iteratively fixes and releases variables from the LP. Variables are fixed when they are deemed contextually promising, and released after some time in order to diversify the search. In the context of this paper, heuristic column generation is used for obtaining an LP-solution: the subproblem is solved by an iterated local search (ILS) procedure.

In order to validate the proposed heuristic algorithm, we apply it to the periodic vehicle routing problem. This is a very well-studied problem for which many heuristic algorithms and an exact approach have already been proposed. It is an extension of the well-known VRP, and takes into account a given planning horizon. Each customer must be visited a required number of times along the planning horizon. In particular, a set of valid day combinations is given for each customer. In addition each customer has a given demand of product, that must be satisfied every time he is visited. A fleet of homogeneous vehicles is also given, each one with the same capacity. The PVRP calls for determining a set of minimum cost routes for each day of the planning horizon, satisfying the following constraints:

1. each customer is visited the required number of times, assigned a combination chosen among a set of appropriate day combinations for this customer, and is delivered the required quantity of product
2. the number of routes on each day does not exceed the total number of available vehicles
3. the capacity of each vehicle is not exceeded.

The PVRP generalizes the VRP since it calls for determining an appropriate day combination for each customer and simultaneously finding a set of minimum cost routes for each day.

The paper is organized as follows: in the next section, we briefly review the main works on PVRP and on periodic traveling salesman problem (PTSP), that is a special case of the PVRP where only one vehicle is available every day; in Section 3 we present the formal description of the problem and provide an integer linear programming (ILP) formulation in a set-covering form with additional constraints, together with the arising subproblem for the solution of the LP-relaxation of the model by column generation; in Section 4 we present the solution method, which consists of solving the LP-relaxation of the SC-formulation in a heuristic way, and of fixing and releasing variables, based on this solution. Experimental results are presented in Section 5 and compared with the state-of-the-art algorithms. Finally, some conclusions are drawn in Section 6.

2. Related work

The PVRP has received much attention in the literature, since it also arises in many real-world applications. A relatively recent survey on the PVRP and its extensions was written by Francis et al. [16]. Due to the difficulty of the problem, most of the works present heuristic approaches, although recently an exact method has been proposed by Baldacci et al. [2]. It is based on a set partitioning integer linear programming formulation of the problem and on three different relaxations used to derive powerful lower bounds.

The first work on the PVRP is proposed by Beltrami and Bodin [3]. Then there are other works by Christofides and Beasley [7], Tan and Beasley [31], Russel and Gribbin [28] and Gaudioso and Paletta [17]. Chao et al. [6] are the first to provide a two-phase heuristic that allows escaping from local optima. They use an integer linear program to assign visit day combinations to customers in order to initialize the system. Moreover, the capacity limit of the vehicles is temporarily relaxed.

In [8] a tabu search (TS) heuristic is proposed that can solve the PVRP, the multi-depot vehicle routing problem (MDVRP) and the periodic traveling salesman problem (PTSP). Two types of neighborhood operators are considered: the first one considers moving a customer from one route to another route on the same day. The second one considers assigning a new visit day combination to a customer. Insertions and removals of customers are performed with the generalized insertion (GENI) operator [18]. Infeasibility with respect to capacity and tour length constraints is allowed during the search process using an adaptive penalty function.

Alegre et al. [1] consider a real world application that arises in periodic pick-up of raw materials for a manufacturer of automobile parts. They propose a Scatter Search to solve this problem and the standard benchmark instances available in the literature. The algorithm is based on a two-phase approach, that first assigns orders to days and then constructs routes for each day.

Furthermore, special implementations for real-world problems are provided by Hadjiconstantinou and Baldacci [19] who propose a heuristic for a multi-depot period vehicle routing problem that arises in the utility sector. Francis et al. [15] solve the PVRP with service choice, where service frequency is a decision of the model. They propose an exact solution method and heuristic variations of it for larger instances. Hemmelmayr et al. [20] propose a variable neighborhood search (VNS) algorithm using three neighborhood structures. Those are moving a sequence of customers from one route to another route,