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# A new heuristic algorithm for cuboids packing with no orientation constraints

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**Abstract**

The three-dimensional cuboids packing is NP-hard and finds many applications in the transportation industry. The problem is to pack a subset of cuboid boxes into a big cuboid container such that the total volume of the packed boxes is maximized. The boxes have no orientation constraints, i.e. they can be rotated by 90° in any direction. A new heuristic algorithm is presented that defines a conception of caving degree to judge how close a packing box is to those boxes already packed into the container, and always chooses a packing with the largest caving degree to do. The performance is evaluated on all the 47 related benchmarks from the OR-Library. Experiments on a personal computer show a high average volume utilization of 94.6% with an average computation time of 23 min for the strengthened A1 algorithm, which improves current best records by 3.6%. In addition, the top-10 A2 algorithm achieved an average volume utilization of 91.9% with an average computation time of 55 s, which also got higher utilization than current best records reported in the literature.


*Keywords:* Cuboids packing; Container loading; Heuristic algorithm; Caving degree

## 1. Introduction

We consider an important packing problem in the three-dimensional (3D) Euclidean space—cuboids packing. The problem is known to be NP-hard and finds many applications in the transportation industry, e.g. container loading and shipment. It is to orthogonally pack a subset of cuboid boxes into a single cuboid container with fixed dimensions, such that the total volume of the packed boxes is maximized. Here boxes can be rotated by 90° in any direction before they are packed. This complicates the problem since the number of possible packing is now a factor of $6^n$ larger for $n$ boxes. Yet, it may allow better solutions, and it is easy to do some modification to make it respecting the constraints on orientation.

Various heuristics have been suggested for solving this problem. Basic methods are wall building [1–4], layering [5], guillotine cutting [6,7], block arrangement [8–10], etc. Then, tree-search [3,10], tabu search [8], genetic search [9] and other local search methods are incorporated to improve the solution's quality. The wall building approach, first introduced by George and Robinson [1], is most commonly used for its efficiency and high quality. In [4], Lim et al. made progress on the wall building approach by allowing every surface of the container to be a wall from which to obtain a multi-faced buildup (MFB). MFB combined guillotine cutting for spare space and was not concern whether

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packed boxes form a flat layer or not. Furthermore, they used a look-ahead strategy on the MFB (MFB_L) by reserving more than one good solution at each packing step. The results of MFB_L algorithm on the well-known OR-Library [11] are about 3% higher than Bischoff's work in [2].

We proposed a new heuristic approach for the packing problem. The inspiration is from an old adage "Gold corner, silver side and grass belly" for Chinese Weiqi that condensed human's experience and wisdom formed in the last thousand years. The adage indicates that corner worth most while center worth dirt on the chessboard. We improved the idea with "Diamond cave" for the cuboid packing problem. Based on Prof. Huang's work for the two-dimensional rectangular packing [12], we define a conception of caving degree to judge how close a packing box is to the boxes filled in already and use backtracking strategy to improve the solution's quality. Experiments on all the 47 related benchmarks from the OR-Library show that we are able to achieve an average packing utilization of 94.6%, which improves current best result reported in the literature by 3.6%.

## 2. Problem definition

In this section, we formulize the problem's definition and prove its computability.

Given a container with dimensions $(L, W, H)$ and a set of boxes (items) $S=\{(l_1, w_1, h_1), \ldots, (l_n, w_n, h_n)\}$. Without loss of generality, suppose all the variables are plus integer numbers. Consider the container embedded into a 3D Cartesian reference frame. The lower-left-near corner coincides with the origin and the upper-right-far corner coincides with point $(L, W, H)$, as Fig. 1 shows. Let $\delta_i \in \{0, 1\}$ denotes whether item $i$ be packed into the container. If item $i$ is packed into the container, let $(x_{i1}, y_{i1}, z_{i1})$ and $(x_{i2}, y_{i2}, z_{i2})$ denote the coordinates of its lower-left-near corner and upper-right-far corner, respectively. Then, the problem can be formulized as follows:

$$\begin{array}{ll} \max & \sum_{i=1}^{n} l_i w_i h_i \delta_i \\ \text{s.t.} & (1)\ (x_{i2}-x_{i1}, y_{i2}-y_{i1}, z_{i2}-z_{i1}) \in \{(l_i, w_i, h_i), (w_i, l_i, h_i), (l_i, h_i, w_i), (h_i, l_i, w_i), (h_i, w_i, l_i), (w_i, h_i, l_i)\}, \\ & (2)\ \max(\max(x_{i1}, x_{j1})-\min(x_{i2}, x_{j2}), \max(y_{i1}, y_{j1})-\min(y_{i2}, y_{j2}), \max(z_{i1}, z_{j1})-\min(z_{i2}, z_{j2}))\delta_i\delta_j \geqslant 0, \\ & (3)\ 0 \leqslant x_{i1} < x_{i2} \leqslant L, \quad 0 \leqslant y_{i1} < y_{i2} \leqslant W, \quad 0 \leqslant z_{i1} < z_{i2} \leqslant H, \\ & (4)\ \delta_i \in \{0, 1\}, \end{array}$$

where $i, j = 1, 2, \ldots, n$ and $i \neq j$.

Constraint (1) implies that each packed item is placed with its dimensions aligned with the coordinate axes. So item $i$ has six orientations whose dimensions on $x$-, $y$-, and $z$-axes are $(l_i, w_i, h_i)$, $(w_i, l_i, h_i)$, $(l_i, h_i, w_i)$, $(h_i, l_i, w_i)$, $(h_i, w_i, l_i)$, and $(w_i, h_i, l_i)$, with their orientation number from 1 to 6, respectively, as Fig. 2 shows. Constraint (2) implies that there is no overlapping between any two packed items; Constraint (3) implies that each packed item is completely in the container. The objective is to find a feasible assignment for variables $\delta_i$, $(x_{i1}, y_{i1}, z_{i1})$, and $(x_{i2}, y_{i2}, z_{i2})$ $(i = 1, 2, \ldots, n)$ that maximizes the container's volume utilization, i.e. minimize the container's wasted space.

**Theorem 1.** *The cubiod packing problem is computable.*

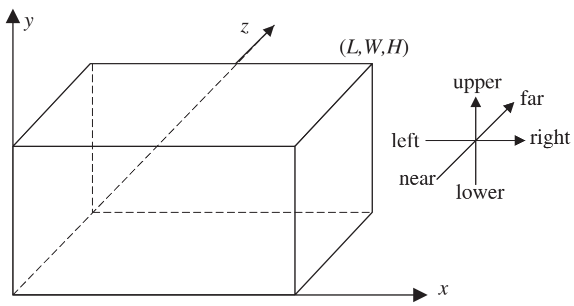


Fig. 1. Coordinate system.