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Interfaces with Other Disciplines

Heuristic algorithms for the multi-depot ring-star problem

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ABSTRACT

In this paper, we consider the problem of designing urban optical networks. In particular, given a set of telephone exchanges, we must design a collection of *ring-stars*, where each ring-star is a cycle composed of a telephone exchange, some customers, some *transition points* used to save routing costs and customers not on the cycle connected to the cycle by a single edge. The ring topology is chosen in many fiber optic communication networks since it allows to prevent the loss of connection due to a single edge or even a single node failure. The objective is to minimize the total cost of the optical network which is mainly due to the excavation costs. We call this problem Multi-Depot Ring-Star Problem (MDRSP) and we formulate it as an optimization problem in Graph Theory. We present lower bounds and heuristic algorithms for the MDRSP. Computational results on randomly generated instances and real-life datasets are also presented.

1. Introduction

In this paper, we present a new optimization problem in graph theory that arises in the design of urban optical networks. The problem is to design a set of *ring-stars*, where each ring-star is a cycle composed of a telephone exchange, some customers, possible points *transition points* that could be used to save routing costs, and customers not on the cycle connected to the cycle by a single edge. Our study was motivated by the interest of a major Italian telecommunication company in the analysis and solution to this type of problem. The cycle (or ring) topology is chosen in many fiber optic communication networks since it prevents the loss of connection due to a single edge or even a single node failure. Indeed, let us suppose that a customer is currently communicating with other customers using the portion of the cycle that goes clockwise from the telephone exchange to it. If an edge (or a node) of this cycle portion fails, then it is possible to re-route the information to/from the customer by using the counter-clockwise portion of the cycle. Each cycle is actually implemented by means of a single cable that contains a bundle of optical fibers (100 fibers in the real-world application that inspired this work). Each customer site receives two fibers: the first one is used for clockwise communication and the second one for counter-clockwise communication. In an urban environment the cable is inserted in pipes laid under the streets. It is not so rare that the pipe and the cable are inadvertently broken during street works. If the network does not restore communication some customers remain disconnected until the cable is repaired. This risky situation is not acceptable for business customers that need to communicate without any interruption. The ring topology is therefore used to guarantee continuous communication service to the customers. The total cost of the optical network depends on several elements as the fibers and the devices needed for communication and restoration, but the main cost is due to the excavations needed to lay down the pipes. Therefore, the telecommunication companies try to reduce such costs as much as possible. Suppose that all customers but one are located on a "perfect" circle. If the distance between this customer and the circle is small (typically under 200 meters) then the network can be implemented with a single cycle plus a single edge that starts from the cycle and goes directly to the outside customer. This means that the cable arrives to the intersection of the circle with the edge, then goes to the customer using the edge, returns to the circle using the same edge, and finally continues along the circle. On the edge the cable is inserted into a single pipe, reducing in this way the excavation costs. If this pipe is broken, the "outside" customer is completely disconnected from the network, while the other customers can use the re-routing technique to continue to communicate. Due to the short length of the edge, this possible failure (and the corresponding loss of service of a unique customer) is considered by the company to be less expensive than the excavation. The use of a single pipe for connecting some customer to the main ring-star, has been used in Italy in several practical applications. The network topology is thus made up of a set of cycles, each containing the telephone exchange, a certain number of customers, some transition points and a number of short edges that go from the cycle to "outside" customers, so the name "ring-star". The cycles must be node-disjoint to ensure that a node failure will only

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affect a single cycle. Note that this constraint has been imposed by the company to reduce the complexity of the repairing operations and the resources necessary to restore the ring-stars, but it is not necessary to ensure connectivity to the customers. In addition, the total number of customers served by a ring-star is limited by the number of fibers in a cable. One hundred fibers can serve 50 customers at most, so typically we are required to design ring-stars with 40–45 customers in order to leave a residual capacity for possible future expansion. Note that, although the main objective function of the problem that inspired this work is to reduce the excavation costs, we will use a more general cost structure (see Section 3) that allows to model a wider class of problems and, in particular, problems where the cycle and the single edges of a ring-star are implemented with different technologies (e.g., optical fibers for the cycle and copper cables for the single edges).

The remainder of this paper is organized as follows. Section 2 gives a formal description of the problem and briefly reviews other works on closely related problems. Section 3 presents a mathematical formulation of the problem together with the description of a procedure for computing lower bounds. Heuristic algorithms for the single telephone exchange case are presented in Section 4, while heuristic algorithms for the general case are presented in Section 5. Section 6 of the paper gives the computational results on two sets of test problems. The first problem set is composed of randomly generated instances involving up to 300 customers and 6 depots. The second one is composed of two real-world instances arising in the design of urban optical networks for Modena and Reggio Emilia Italian towns of the Emilia Romagna region. Finally, conclusions are reported in Section 7.

2. Problem description

We start by defining the problem with a graph theory model. We use a mixed graph in which customers, transition points and a set of telephone exchanges are associated with *nodes,* while the links between the nodes are unoriented weighted *edges* or oriented weighted *arcs*. Using an analogy to the routing literature, we will call the telephone exchanges *depots*, so we call our problem the *Multi-Depot Ring-Star Problem* (MDRSP). MDRSP consists of designing: (i) a set of node-disjoint (with the exception of the node representing the depot) simple cycles, each one passing through one and only one depot and through some other nodes representing customers or transition points; and (ii) a set of direct connections from each non-visited customer to a node of a cycle. The total number of customer nodes visited by or connected to a cycle is bounded by an upper *capacity* limit Q and, in addition, the total number of cycles emanating from each depot is limited by an upper limit. The objective is to minimize the cost of the cycles plus the costs of the customer connections. More precisely, MDRSP is as follows. Consider a mixed graph $G = (V, E \cup A)$, where $V = D \cup V'$ is the node set, $E = \{\{i,j\} : i,j \in V, i \neq j\}$ is the edge set, and A is the arc set. Node set D represents the depots. Node set V' is partitioned into two subsets: U the set of customer and W the set of transition points (also named *Steiner nodes*). For each customer $i \in U, C_i \subseteq V'$ denotes a subset of nodes "close" to i, called *connection set* (see below for a formal definition). The arc set $A = \{(i,j) : i \in U, j \in C_i\}$ contains the possible direct *connections* between each customer an the other nodes. A customer i outside a cycle can be *connected* to the cycle using one of the arcs $(i,j) \in A$, where j is a node of the cycle. Node i is called the *tail* of arc (i,j) and node j the *head*. Finally, in order to simplify the model,we will assume that $i \in C_i$ so that we can formally connect a customer to itself when it is on a cycle. Each edge $e \in E$ is associated with a non-negative *routing* cost c_e, while each arc $(i,j) \in A$ is associated with a non-negative *connection* cost d_{ij} (with $d_{ii} = 0$ for all $i \in U$). Given a customer $i \in U$, the set C_i is defined as $C_i = \{j \in V' : d_{ij} \leqslant s\}$, for a given target value s.

Fig. 1 shows a feasible MDRSP solution for an instance with $|V'| = 24, |U| = 12, |W| = 12, Q = 6$ and involving a single telephone exchange (i.e., $|D| = 1$). The transition points are represented by circles while the customers by triangles. The telephone exchange is represented by a black square. The continuous lines represent the routing edges (set E) and the dashed lines represent the connection arcs (set A).

Given a subset $E' \subset E$, let $G' = (V(E'), E')$ denote the induced subgraph of graph G. A *ring-star* is a triplet (k, E', A') where: (i) $k \in D$; (ii) $E' \subset E$ is a set of routing edges forming a simple cycle containing node k; and (iii) $A' \subseteq A$ is a set of connections between customers and nodes of $V(E')$. We say that a customer i is *assigned* to a ring-star if it is either visited by the cycle (i.e. $i \in V(E')$ and $(i,i) \in A'$) or it is connected to a node of the ring-star (i.e. there exists a node $j \neq i$ such that $(i,j) \in A'$). The ring-star is *feasible* if the number of customers assigned to it does not exceed Q. The *cost* of the ring-star is the sum of the routing costs of the edges in E' plus the sum of the connection costs of the arcs in A'. For each depot $k \in D$, the maximum number m_k of ring-stars emanating from it is known and given as an input parameter. (A trivial assumption is that $Q\sum_{k \in D} m_k \geqslant |U|$.)

The aim of MDRSP is to design for each depot $k \in D$, at most m_k ring-stars so that each customer is assigned to exactly one ring-star, each transition point is visited at most once and the sum of the ring-star costs is minimized. When $|D| = 1$, according to the notation introduced in [1], we call the problem the Capacitated m-Ring-Star Problem (CmRSP), where m denotes the maximum number of ring-stars emanating from the unique telephone exchange.

The MDRSP is $\mathcal{NP}$-hard in the strong sense, since the well known *Travelling Salesman Problem* (TSP) is the special case in which $|D| = 1, W = \emptyset, Q = |U|, m_0 = 1$ and $C_i = \{i\}, \forall i \in U$. To the best of our knowledge, only the single-depot version of MDRSP has been studied before by Baldacci, Dell'Amico and Salazar [1], who proposed two different integer programming formulations, compared the corresponding linear programming relaxations, designed valid inequalities and separation procedures, and finally implemented an overall branch-and-cut algorithm. This algorithm solved to optimality instances with up to 100 customers. Since real-life instances have much more customers and depots (see the real-world instances described in Section 6.2), the effort in studying and developing good heuristic algorithms is justified.

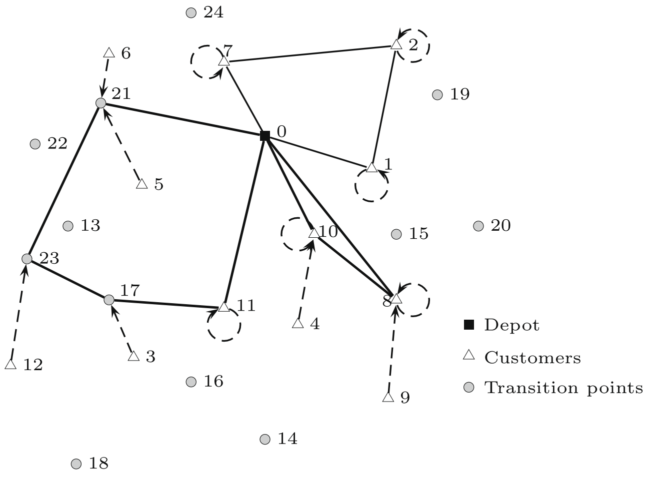

Fig. 1. Example of feasible solution for a single-depot instance.