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A stochastic Nash equilibrium portfolio game between two DC pension funds

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HIGHLIGHTS

- Study stochastic Nash equilibrium portfolio game of two DC pension funds.
- Derive closed-forms of the Nash equilibrium portfolio strategies.
- Give numerical analysis to investigate evolutions of the Nash equilibrium strategies.

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ABSTRACT

In this paper, we study the stochastic Nash equilibrium portfolio game between two pension funds under inflation risks. The financial market consists of cash, bond and two stocks. It is assumed that the price index is derived through a generalized Fisher equation while the bond is related to the price index to hedge the risk of inflation. Besides, these two pension managers can invest in their familiar stocks. The goal of the pension managers is to maximize the utility of the weighted terminal wealth and relative wealth. Dynamic programming method is employed to derive the Nash equilibrium strategies. In the end, a numerical analysis is presented to reveal the economic behaviors of the two DC pension funds.
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1. Introduction

Pension fund management has attracted more and more attention and becomes a popular subject in recent years. Pension fund can be viewed as a saving vehicle before retirement to ensure retirement income and thus has great importance. Therefore, an efficient pension fund is very important for the individual and society, and there are many works studying the management of pension fund. Usually, there are mainly two kinds of pension fund, classified by how the contribution and benefit are set in the plan. The first one is defined contribution (DC) pension plan, which involves a fixed contribution rate before retirement. However, the benefit is not fixed for DC pension plan but is determined by the investment

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performance of the pension plan. The second one is the defined benefit (DB) pension plan. In the DB pension plan, the benefit is fixed while the contribution should be adjusted continuously to keep balance. Thus, the sponsors will undertake the risks in the DB pension plan. Recently, in contrast with the DB pension plan, the DC pension plan has attracted more attention and develops more and more fast.

In the DC pension plan, the portfolio of the plan before retirement is very important to ensure the benefit after retirement. There are many works investigating the optimal portfolios before retirement to maximize the expectation of utility of terminal wealth. Vigna and Haberman (2001) firstly studied the optimal DC pension plan in a discrete model. In their further work (cf. Haberman and Vigna, 2002), they also paid attention to the risk measures for the DC plan. Boulier et al. (2001) established an efficient continuous financial model for the DC plan with stochastic interest rate. They successfully derived the optimal investment strategies in a portfolio insurance problem for the pension manager. Later, Deelstra et al. (2003) considered a more general financial model and solved the optimization problem by introducing auxiliary processes and martingale method.

Since the accumulation phase of a DC pension plan is often very long, about 20–40 years, the risks in the market will influence the pension fund heavily, mainly *the risks of inflation and interest rate*. Zhang et al. (2007) firstly analyzed the economic behavior of a DC pension manager under inflation. After that, Han and Hung (2012) obtained the optimal allocations of a pension plan with CRRA utility preference under inflation and interest rate risks. Yao et al. (2013) initially solved the mean–variance problem for a pension manager with inflation risk. Dynamic programming method was applied to obtain the efficient frontier in their work. Wu et al. (2015) derived the closed form solutions for the pension manager under inflation risk in a time-inconsistent mean–variance framework. Moreover, the risks of mortality and contribution for a pension manager were introduced in Yao et al. (2014).

Besides the risks of the market, some papers also consider the risks of stock for a pension plan. The stock price in the previous papers follows a geometric Brownian motion, which does not characterize the features of stock well. Gao (2009) characterized the stock price by a constant elasticity of variance (CEV) model. The Legendre transform and dynamic programming method were combined in his work to derive the optimal strategies both before and after retirement. Guan and Liang (2014) established a financial model for DC pension manager under interest rate and volatility risks. The optimization goal in the paper was to maximize the CRRA utility of terminal wealth over an annuity guarantee. Besides, in more work (cf. Guan and Liang, 2015), a thorough research was conducted for the DC pension plan under stochastic interest rate and mean-reverting returns.

In most papers, a pension manager is only concerned with the maximization of the expectation of the utility of terminal wealth. However, in the real market, there exists competition between different pension managers. The pension manager is concerned about relative performance and considers the terminal wealth and relative wealth at the same time. Therefore, if the pension manager intends to behave better than the other manager to attract more attention, it is more realistic to take into account the other manager's economic behavior. Some existing works are concerned with the competition between different managers. The goal of the manager is to maximize the expectation of the utility of weighted terminal wealth and relative wealth. However, there may not exist optimal investment strategies achieving the managers' goal at the same time, hence they often search the Nash equilibrium strategies for different competitors. Browne (2000) firstly solved the problem of portfolio games for two investors and derived the equilibrium strategies for some specified games. Meanwhile, Bensoussan and Frehse (2000) studied the regularity condition for the existence of Nash strategies in a stochastic games among N players by dynamic programming method. Basak and Makarov (2014) investigated the case of competition between two investors. In their work, the investor was cared about the relative wealth when it was above a level. They beautifully employed the martingale method to derive the Nash equilibrium strategies. Later, they (cf. Basak and Makarov, 2013) explored the relation between competition and asset specialization. The competition between two insurance companies was studied in Bensoussan et al. (2014). In their work, proportional reinsurance could be purchased and one insurance company intended to maximize the utility of the difference between the insurance company and the other one. They also applied the dynamic programming method to derive the Nash equilibrium strategies. Later, Meng et al. (2015) extended the model to the case when the surplus process of the insurer was characterized by a nonlinear (quadratic) risk control process.

In this paper, we consider the competition between two DC pension managers. One pension manager will try to have a better performance than the other manager to attract more attention. In the market, since the time of a DC plan is often long, we take into account the influence of inflation risk. The financial market contains cash, bond and two stocks. The bond is related to the price index and can help hedge the risk of inflation. However, since the two managers are willing to invest in their familiar stocks, the stocks the two managers invest in are not the same. The goal of the pension manager is to maximize the utility of his terminal wealth and the relative wealth w.r.t. the other pension manager. So we need to solve two different optimization problems. However, since there hardly exist optimal investment strategies for these two problems at the same time, we search the Nash equilibrium strategies by dynamic programming method, i.e., each manager is assumed to know the equilibrium strategy of the other manager, and no one will change his own strategy. In the end of the paper, we present the numerical analysis to show the evolutions of the Nash strategies and wealths.

The rest of this paper is organized as follows: The financial market and the structure of the pension fund are presented in Section 2. In Section 3, we study the competition between two pension managers and derive the Nash equilibrium strategies. Section 4 shows the Nash equilibrium strategies and evolution of the wealths. Section 5 is a conclusion.

2. The financial market and the pension management

In this section, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbf{P})$ be a filtered complete probability space. $\mathcal{F}_t$ represents the information of the market available before time t. Besides, $[0, T]$ is a fixed time horizon and the pension managers can adjust their investment strategies continuously within $[0, T]$. In what follows, we assume that all the processes are well-defined and adapted to $\{\mathcal{F}_t, t \in [0, T]\}$.

2.1. The financial market

In this paper, we consider the inflation risk for the pension funds, which can help hedge the risk of inflation in the long run of a pension fund. The risks of inflation and the financial market are presented in this section. In fact, there exist many treasury inflation-protected securities in the market to hedge the risk of inflation and we introduce a particular asset named inflation-indexed zero coupon bond in our market. Thus, the financial market in our work consists of cash, treasury inflation-protected securities and two stocks. The price of the risk-free (i.e., cash) asset $S_0(t)$ is the following:

$$\frac{\mathrm{d}S_0(t)}{S_0(t)} = r_n(t)\mathrm{d}t, \quad S_0(0) = S_0, \tag{2.1}$$

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