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Do multiple Nash equilibria in Markov strategies mitigate the tragedy of the commons?

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Abstract

In a seminal paper, Dockner and van Long [1993. International pollution control: cooperative versus non-cooperative strategies. Journal of Environmental Economics and Management 25, 13–29] argue that nonlinear strategies allow to protect the commons compared with the linear (and singular) strategy. This paper shows that the existence of multiple equilibria depends on preference characteristics and that multiple equilibria need not foster conservation. In particular, if the (absolute) elasticity of marginal utility is decreasing, then multiple equilibria aggravate the tragedy of the commons compared with the singular (or saddlepoint) strategy.

JEL classification: C61; D62

Keywords: Dynamic games; Conservation; Multiple equilibria

1. Introduction

Players $i = 1, \ldots, n$ choose their emissions $x_i(t)$ at time t maximizing their individual net present value (using the identical discount rate $r > 0$),

$$\max_{\{x_i(t)\}} J_i \equiv \int_0^\infty \left[u(x_i(t)) - \frac{c}{2} X^2(t) \right] e^{-rt} \, dt. \quad (1)$$

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The instantaneous utility consists of the individual benefit from emissions (u) minus the (quadratic, $c>0$) external costs from the pollution stock X, which accumulates over time according to

$$\dot{X}(t) = \sum_{i=1}^{n} x_i(t) - \delta X(t), \quad X(0) = X_0, \ \ \delta>0. \tag{2}$$

Assuming linear-quadratic benefit and symmetric Nash equilibria, Dockner and van Long (1993) show that a play in nonlinear Markov strategies instead of in the linear strategy mitigates and with sufficient patience even eliminates the tragedy of the commons (in the long run). This surprising and optimistic result has triggered many discussions. Rubio and Casino (2002) stress the restricted support of equilibria in nonlinear strategies. Rowat (2000) demands global strategies and Rowat (2006) includes sufficiency conditions, yet all these reconsiderations confirm the basic conclusion of Dockner and van Long (1993) that 'nonlinear strategies do better than the linear or singular strategy and sufficient patience allows to obtain the first best'. Dutta and Sundaram (1993) and Dockner and Nishimura (2006) analyze discrete time versions.

However, and this is the purpose of this short paper, conservation is not an intrinsic property of nonlinear or multiple equilibria (better and more generally characterized as 'non-singular'), but is a consequence of the preferences, more precisely, of the derivative of the elasticity of marginal utility,

$$\sigma(x) \equiv -\frac{u''(x)x}{u'(x)}, \tag{3}$$

which is increasing, $\sigma'<0$, for a linear-quadratic specification of u. If in contrast $\sigma'<0$, multiple equilibria worsen the tragedy of the commons compared with the singular (or saddlepoint) equilibrium strategy, which is the linear strategy in the case of a linear-quadratic differential game. And it is impossible in this case to approach the first best steady state no matter how patient the players are. The common denominator with the claim in Dockner and van Long (1993) is that multiple equilibria are confined to the domain of small values of σ, i.e. where the (relative) marginal gain from incremental emissions is small.

2. Model and assumptions

In order to verify the claims above in a compact manner, the same assumptions are made and similar techniques are applied as in the related literature, and a specification of u with $\sigma'<0$ is assumed although the same conclusion holds for any utility with a decreasing elasticity of marginal utility. Following Dockner et al. (2000, p. 85) (also used in Rowat, 2006), the maximization of player i's payoff J_i, which depends on the own $x_i = \phi_i$ and the opponents' strategies $\Phi^{-i} = \{x_j = \phi_j \mid j \neq i\}$ and on the initial condition X_0, can be written as an optimal control problem,

$$V_i(X_0) \equiv \max_{x_i} J_i, \quad i = 1, \ldots, n \tag{4}$$