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Efficient Markov perfect Nash equilibria: theory and application to dynamic fishery games

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Abstract

In this paper, we present a method for the characterization of Markov perfect Nash equilibria being Pareto efficient in non-linear differential games. For that purpose, we use a new method for computing Nash equilibria with Markov strategies by means of a system of quasilinear partial differential equations. We apply the necessary and sufficient conditions derived to characterize efficient Markov perfect Nash equilibria to dynamic fishery games.

JEL classification: C73; Q22

Keywords: Differential games; Markov perfect Nash equilibria; Pareto optimum; Quasilinear partial differential equations; Fishery management

1. Introduction

The interdependence between the decisions of the agents is well known in many areas of economics. Game theory is a useful tool to study these situations involving several decision makers. An interesting question is to determine whether the non-cooperative solution presents the property of Pareto efficiency. The question is

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whether a cooperative behavior can emerge without any binding agreements between the agents. The efficiency of Nash equilibrium is a very uncommon feature, but at the same time a very desirable property, since it makes the efficient solution self-enforcing. If, moreover, the Nash equilibrium is subgame perfect, the efficient solution can be enforced for any initial condition during the course of the game.

If a priori efficiency is not at hand, a standard method to implement cooperative solutions by means of non-cooperative play is to construct efficient Markov perfect Nash equilibria (MPNE) based on trigger strategies, as in Haurie and Pohjola (1987) or Tolwinski et al. (1986). Within the limits of differential games theory, this approach presents technical problems because trigger strategies are in general discontinuous and needs the introduction of memory strategies for the players. These strategies are based on all past information of the game evolution to the current time, and as a consequence, this kind of strategies are non-Markovian. Another option is the use of incentive strategies, establishing the efficient solution as an incentive equilibrium (see, for example, Ehtamo and Hämäläinen, 1989, 1993; Jørgensen and Zaccour, 2001).

It is important to note that the attainment of the cooperative solution as a Nash equilibrium of a non-cooperative game will depend on what set of strategies is available to the players. For that reason, this paper proposes a new approach that allows to identify games where the MPNE based on Markov strategies is Pareto efficient. This new approach is based on the characterization of MPNE as solutions to a system of quasilinear partial differential equations. This system is obtained from the optimality conditions of the maximum principle. The quasilinear system is fully equivalent to the Hamilton–Jacobi–Bellman system, as pointed out in Rincón-Zapatero et al. (1998). The former system characterizes MPNE directly, whereas the latter characterizes the value functions. However, a quasilinear system is much more amenable than a fully non-linear system of partial differential equations as the Hamilton–Jacobi–Bellman system.

To our knowledge, there are no necessary and sufficient conditions in the literature to be applied to general differential game models that allow to determine whether the feedback Nash equilibrium is or not a Pareto optimum. This paper is devoted to establish a method that allows us to identify the coefficient functions of a rather general differential game in order that the game possesses an efficient MPNE. A particular case, for differential games with unidimensional state and control variables and where each control is a smooth function of the state and time variable, has been studied in Rincón-Zapatero et al. (2000). The importance of the identification of equilibria being Pareto optimum relies on the stability properties of Nash's concept and on the efficiency of the cooperative solution. An extreme case of this type of solutions are the so called absolutely cooperative, which are studied, e.g., in Leitmann (1974).

It is well known that, in general, the use of Markov strategies in non-cooperative games prevents to attain efficient outcomes (see, for example, Dubey, 1986). In fact, the efficiency of Nash equilibrium can be considered a very rare property. However, such a property has been noticed to hold for some fishery games Chiarella et al. (1984). In this paper, the players are restricted to use open-loop strategies, so the