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Games and Economic Behavior 50 (2005) 332–365
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Nash equilibrium and generalized integration for infinite normal form games

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Received 13 June 2001

Available online 10 December 2004

Abstract

Infinite normal form games that are mathematically simple have been treated [Harris, C.J., Stinchcombe, M.B., Zame, W.R., in press. Nearly compact and continuous normal form games: characterizations and equilibrium existence. Games Econ. Behav.]. Under study in this paper are the other infinite normal form games, a class that includes the normal forms of most extensive form games with infinite choice sets.

Finitistic equilibria are the limits of approximate equilibria taken along generalized sequences of finite subsets of the strategy spaces. Points must be added to the strategy spaces to represent these limits. There are direct, nonstandard analysis, and indirect, compactification and selection, representations of these points. The compactification and selection approach was introduced [Simon, L.K., Zame, W.R., 1990. Discontinuous games and endogenous sharing rules. Econometrica 58, 861–872]. It allows for profitable deviations and introduces spurious correlation between players' choices. Finitistic equilibria are selection equilibria without these drawbacks. Selection equilibria have drawbacks, but contain a set-valued theory of integration for non-measurable functions tightly linked to, and illuminated by, the integration of correspondences.
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JEL classification: C62; C72

Keywords: Infinite games; Equilibrium; Nonmeasurable integration; Compactification; Selection equilibria

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doi:10.1016/j.geb.2004.09.007

1. Introduction

A normal form game (nfg), $\Gamma = (S_i, u_i)_{i \in I}$, is specified by a finite player set, I, strategy sets, S_i, $i \in I$, and bounded utility functions, $u_i : S \to \mathbb{R}$, $S := \times_{i \in I} S_i$. This paper develops a theory of Nash equilibrium for nfgs specified at this level of generality. There are no topological or measure theoretic assumptions.

Compact and continuous nfgs are the starting point for the study of infinite games. An nfg is compact and continuous if each S_i is compact and each u_i is jointly continuous. The companion piece to this paper developed the theory of nfgs that are nearly compact and continuous (ncc).

1.1. Games that are nearly compact and continuous

A game Γ is ncc if it is possible to densely imbed each S_i in a compact space, $\widehat{S_i}$, in such a fashion that all of the u_j have jointly continuous extensions to the product $\times_{i \in I} \widehat{S_i}$. A game Γ is *integrable* if each u_i is integrable with respect to all products of finitely additive probabilities. A game Γ is *uniformly finitely approximable* (ufa) if each S_i can be approximated by finite sets using the Fudenberg and Levine's (1983) "most utility difference it can make to anyone" pseudo-metric,

$$d_{\mathcal{U}_i}(s_i, t_i) = \max_{k \in I} \sup_{s \in S} \left| u_k(s \backslash s_i) - u_k(s \backslash t_i) \right|. \tag{1}$$

The companion piece to this paper, Harris et al. (in press), showed that the three conditions, integrability, being ufa, and being ncc, are equivalent. This paper studies nfgs that fail to be integrable, ncc, or ufa, a class that includes the normal forms of most extensive form games with infinite choice sets.

1.2. Extensive form games

Suppose that Γ is the normal form representation of an extensive form game in which player 1 makes a pick s_1 in an infinite set S_1, s_1 is subsequently observed by player 2, who then picks an action a in a set $A = \{a, b\}$, and that player 2's choice of a or b always makes at least a utility difference of at least 1 to some player. Most extensive form games involve at least this much dynamic interaction between players. While it is not at all clear what set of strategies should be considered for player 2, a minimal requirement is that the class of functions, $S_2 \subset A^{S_1}$, constituting player 2's strategy set, must be dense in the product topology.[1]

The denseness implies that for all $s_1 \neq t_1 \in S_1$, there exists an $s_2 \in S_2$ such that $s_2(s_1) \neq s_2(t_1)$, implying that $d_{\mathcal{U}_1}(s_1, t_1) \geqslant 1$. Also, if $s_2 \neq t_2$ iff there exists an s_1 such that $s_2(s_1) \neq t_2(s_1)$ so that $d_{\mathcal{U}_2}(s_2, t_2) \geqslant 1$. The normal form of this game is therefore not ufa. By the cited equivalence result, Γ is neither integrable nor ncc.

[1] This is equivalent to 2's strategies allowing arbitrary patterns of response at all finite subsets of S_1. More explicitly, if F_1 is a finite subset of S_1, then for every vector $x_2 \in A^{F_1}$, there is a strategy in S_2 that agrees with x_2 at the points in F_1.

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