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Purification of Bayes Nash equilibrium with correlated types and interdependent payoffs [☆]

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ABSTRACT

We establish purification results for Bayes–Nash equilibrium in a large class of Bayesian games with finite sets of pure actions. We allow for correlated types and interdependent payoffs and for type-dependent feasible action sets. The latter feature allows us to prove existence and purification results for pure Bayes–Nash equilibria in undominated strategies. We give applications to auctions, global games, and voting to illustrate the usefulness of our results.

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1. Introduction

Purification is a potentially powerful tool for obtaining existence of a pure strategy Bayes–Nash equilibrium (BNE) in games of incomplete information. It ensures, under non-atomicity of the underlying distribution of types and some other regularity conditions, that for every BNE in mixed strategies, there exists an equivalent BNE in pure strategies. Thus, insofar as existence of a BNE in mixed strategies has been established in great generality (cf. Balder, 1988, 2002), it suffices that one verifies the conditions needed for purification for a BNE in pure strategies to exist. Unfortunately, the extra regularity conditions for purification provided in the literature are quite restrictive: in particular, types are required to be independent conditional on a finite environmental state variable, and no correlation or interdependence of payoffs in addition to this environmental state variable is allowed. A further limitation of this approach is that known existence results do not adhere to common refinements used in applied modeling, such as the requirement that players use undominated strategies. We address these issues by proving a purification result that allows for general forms of correlation of types and of interdependence of payoffs, and by establishing existence (and purification) in the class of undominated BNE. We illustrate our results with applications to auctions, global games, and voting with incomplete information.

We consider games of incomplete information among n players who choose from finite sets of pure actions, in which the types of each player i can be decomposed into two components, t_i and u_i . The first is a general component that affects the payoffs of every player; the second is a private-value component that affects only player i 's payoffs and moreover is

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conditionally independent given t_i . We allow the general type components (t_1, \dots, t_n) to be highly correlated (subject only to the standard diffuseness condition of [Milgrom and Weber, 1985](#)), and because of this, our framework encompasses models in which players receive conditionally independent signals of an underlying general state variable in addition to some conditionally independent information (e.g., preference shocks). We provide two results. First, we establish purification, and thereby existence of pure strategy BNE in a large class of models with correlated types and interdependent payoffs. Second, we sharpen the first result by proving that an undominated BNE exists, and that for every such equilibrium, there is an equivalent pure-strategy BNE that also satisfies the refinement. Interestingly, the latter argument relies essentially on our initial purification result: after eliminating undominated pure actions, we obtain a mixed strategy BNE that assigns positive probability only to undominated pure actions, but this mixed BNE may well be dominated. We address this issue by purifying the mixed BNE so that, by construction, players choose only undominated pure actions. Our results deliver pure strategy undominated BNE in applications where players have correlated information that is payoff-relevant for other players, and in particular to applications such as auctions and voting where such BNE are especially important.

Our approach to purification is as follows. Given type (t_i, u_i) , we assume player i mixes over a finite set of pure actions. A mixed strategy σ_i assigns a mixed action to each realization (t_i, u_i) . Integrating out the private component of the type u_i , we obtain the average action $\gamma_i(t_i|\sigma_i) = \int \sigma_i(t_i, u_i) du_i$. Because payoffs are multilinear in mixed actions, the interim payoff function of player i depends on i 's own action and type and on the profile of average actions $\gamma_{-i} = (\gamma_j)_{j \neq i}$. Thus, the set of interim best response actions can be written as $M_i(t_i, u_i; \gamma)$. Now consider the set $\mathcal{G}_i(\gamma)$ of selections from the correspondence $t_i \rightarrow \int M_i(t_i, u_i; \gamma) du_i$ from general types to average best response actions. We show that each fixed point $\gamma^* \in \mathcal{G}(\gamma^*)$ (there is at least one in our framework) corresponds to a class of equivalent BNE; and using the assumption that the private-value components are non-atomically distributed, there is necessarily a pure BNE belonging to this class.¹ Our results for undominated BNE follow from a proposition showing that the set of interim undominated pure actions for a player varies with respect to types in a measurable way. We then incorporate the restriction to undominated pure actions into the players' feasible sets of actions and apply our existence and purification theorem, recalling the important role (described in the previous paragraph) of purification in the existence of an undominated BNE.

There are substantial literatures on purification and pure strategy existence that impose more structure. [Radner and Rosenthal \(1982\)](#) provide an early purification result, also assuming a decomposition of types, but they assume the t_i 's live in a finite set, that the u_i 's are (unconditionally) independent, and that payoff interdependence and correlation of types is at most finite dimensional.² Other results in the purification literature (e.g., [Milgrom and Weber, 1985](#); [Khan et al., 2006](#); [Fu et al., 2007](#); and [Balder, 2008](#)) explicitly assume a finite environmental variable t_0 , and that player types are private values and independent conditional on t_0 . As such, only the special form of correlation and interdependence of payoffs mediated by the finite state variable t_0 is allowed. In contrast, we allow the general type components (t_1, \dots, t_n) to be highly correlated (subject to diffuseness) and arbitrarily payoff relevant for all players, which permits a general space of environmental states. It should be noted that there are many recent purification results that rely on extremely diffuse environments (e.g., [Podczeck, 2009](#) and [Wang and Zhang, 2010](#)) and obtain purification results for games with general action spaces. As with the standard purification literature, this agenda does not allow for general forms of correlation of types or interdependence of payoffs, and its practical usefulness is limited by the assumption of super non-atomicity. Moreover, as recently shown by [Greinecker and Podczeck \(2013\)](#), purification in extremely diffuse environments is spurious as the resulting purified strategies still involve mixing. There is also a sizable literature on existence of pure strategy BNE based on modularity ideas (cf. [Athey, 2001](#); [McAdams, 2003](#); [Van Zandt and Vives, 2007](#); [Van Zandt, 2010](#); [Reny, 2011](#)), including recent work by [de Castro \(2012\)](#), who uses a decomposition of types that is different from ours.³ As our applications show, our approach ensures existence of pure strategy BNE in models previously studied in this literature without the modularity restrictions needed to deduce equilibria with a monotone structure.

Less is known about existence of undominated BNE in pure or mixed strategies, and in fact simple examples (cf. [Simon and Stinchcombe, 1995](#)) illustrate that infinite normal form games may not admit undominated equilibria. The normal form of the Bayesian games we study are infinite games, but the product structure on type sets and non-atomicity of the private-value component allow us to circumvent existence counterexamples. Despite terminological similarity, our results are unrelated to those of [Le Breton and Weber \(1997\)](#) and [Balder \(2003\)](#), who consider large non-atomic games and show existence of an equilibrium that is "undominated" in the sense that it is not Pareto dominated by any other equilibrium.

The paper is organized as follows. Section 2 presents a number of applications to illustrate the practical usefulness of our results. In Section 3, we present the product Bayesian game framework, and we state our results in Section 4. Proofs of the main results are in Section 5.

¹ The techniques used here are similar to those used by [Duggan \(2012\)](#) in establishing existence of stationary Markov perfect equilibria in a class of stochastic games.

² The use of types with more than one component is a common feature in the purification literature; see, e.g., [Khan et al. \(2006\)](#), [Fu et al. \(2007\)](#), or [de Castro \(2012\)](#).

³ As a sufficient condition for his richness condition, [de Castro \(2012\)](#) uses a type decomposition in which one component is a private-value type that determines a player's preferences, and the other component is a payoff-irrelevant belief type; moreover, he assumes a partial ordering of preference types and a separability condition on belief types. Our decomposition assumes a private-value component, but the remaining component is completely general, and we do not impose any ordering or separability conditions.

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