European Journal of Operational Research 252 (2016) 728–736

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A heuristic algorithm for container loading of pallets with infill boxes

Liu Sheng[a,b,*], Zhao Hongxia[a,b], Dong Xisong[a,b], Cheng Changjian[a,b]

[a] State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China
[b] Qingdao Academy of Intelligent Industry, Qingdao 266000, China

ARTICLE INFO

Article history:
Received 26 August 2013
Accepted 16 January 2016
Available online 25 January 2016

Keywords:
Packing
Container loading
Heuristic algorithm
Tree search
Greedy algorithm

ABSTRACT

We consider the container loading problem that occurs at many furniture factories where product boxes are arranged on product pallets and the product pallets are arranged in a container for shipments. The volume of products in the container should be maximized, and the bottom of each pallet must be fully supported by the container floor or by the top of a single pallet to simplify the unloading process. To improve the filling rate of the container, the narrow spaces at the tops and sides of the pallets in the container should be filled with product boxes. However, it must be ensured that all of the infill product boxes can be entirely palletized into complete pallets after being shipped to the destination. To solve this problem, we propose a heuristic algorithm consisting of a tree search sub-algorithm and a greedy sub-algorithm. The tree search sub-algorithm is employed to arrange the pallets in the container. Then, the greedy sub-algorithm is applied to fill the narrow spaces with product boxes. The computational results on BR1–BR15 show that our algorithm is competitive.

1. Introduction

Cutting and packing (Dyckhoff & Finke, 1992; Wäscher, Haußner, & Schumann, 2007) are two classic combinatorial optimization problems. Cutting problems address the best possible utilization of materials, such as wood, steel and cloth, whereas packing problems address the best possible capacity use of packing space. The effective use of material and transport capacities is of great economic importance in production and distribution processes. It also contributes to the economical utilization of natural resources.

According to Wäscher et al. (2007), cutting and packing problems have an identical structure in common. They can be summarized as follows:

First, a set of large objects and a set of small items are given. The large objects and the small items are defined exhaustively in one, two, three or an even larger number of geometric dimensions. Select some or all of the small items, group them into one or more subsets and assign each of the resulting subsets to one of the large objects such that the geometric condition holds, *i.e.*

- all small items of the subset lie entirely within the large object, and
- the small items do not overlap,

and a given objective function is optimized.

Container loading problems are sub-problems of the cutting and packing problems. In Bortfeldt and Wäscher (2013), container loading problems are interpreted as geometric assignment problems, in which three-dimensional small items (called cargo) have to be assigned (packed into) to three-dimensional, rectangular (cubic) large objects (called containers) such that a given objective function is optimized and two basic geometric feasibility conditions hold, *i.e.*

- all small items lie entirely within the container, and
- the small items do not overlap.

In this paper, we consider a container loading problem for pallets with infill boxes (CLPIB), which is a special container loading problem case. Given an empty rectangular container and m rectangular product pallets, we determine a subset of pallets with maximal volume that can be placed in the container. Each product pallet contains a single type of rectangular product boxes. Some product pallets can be divided into product boxes for transportation. The product boxes can be placed in the narrow spaces between the pallets and the container after a container is filled with pallets. After the container is shipped to its destination, these product boxes should be exactly palletized onto their original

* Corresponding author at: State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China. Tel.: +86 15910634676.
E-mail address: liusheng7801@163.com, sheng.liu@ia.ac.cn (L. Sheng).

http://dx.doi.org/10.1016/j.ejor.2016.01.025
0377-2217/

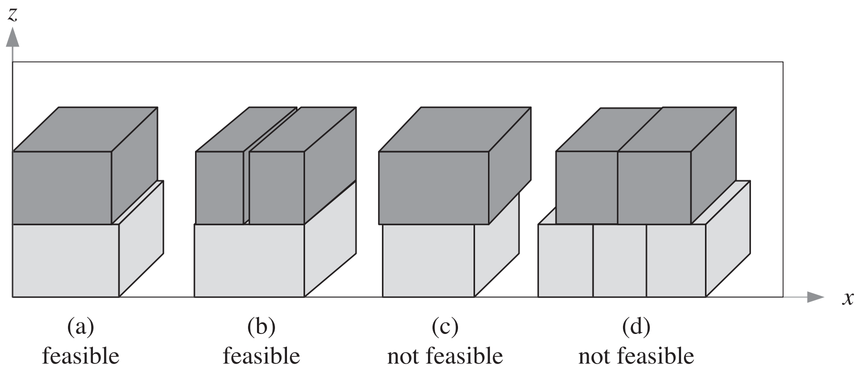

Fig. 1. Four pallet arrangements in a container.

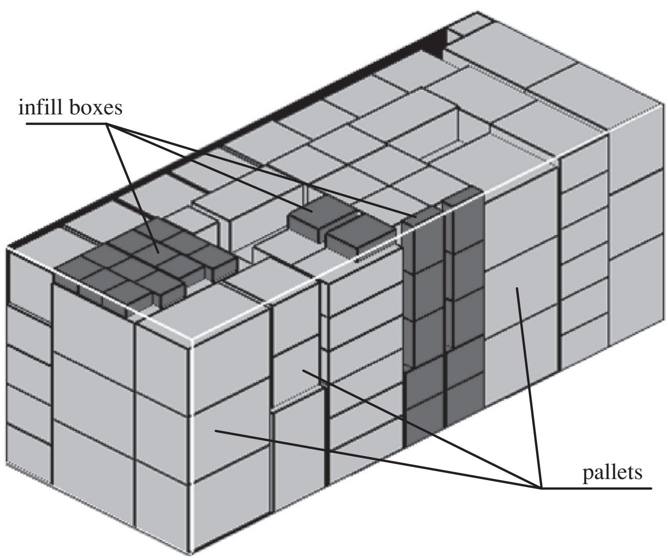

Fig. 2. An example solution for CLPIB.

pallets. The pallets must be placed with their bottoms parallel to the container, whereas the product boxes can be placed in all six orthogonal orientations. Additionally, the bottoms of each pallet must be fully supported by the container floor or by the top of a single pallet to simplify the unloading process and to ensure the stability of the pallets. The product boxes can be easily fastened on the pallets by adhesive tapes because they are relatively small and light. Thus they do not need to be fully supported.

Fig. 1 lists four pallet arrangements in a container. The arrangements in Fig. 1(a) and (b) are feasible, whereas the arrangements in Fig. 1(c) and (d) are not feasible.

An example solution for CLPIB is illustrated in Fig. 2.

The rest of this paper is divided as follows: Section 2 provides an overview of the literature, Section 3 presents the approach for CLPIB, Section 4 describes the computational experiments and presents the results, and Section 5 summarizes the paper.

2. Literature overview

Bortfeldt and Wäscher (2013) introduced a scheme to categorize the constraints of loading a container for the first time and found that the existing approaches have limited practical value because they do not pay sufficient attention to the constraints encountered in practice. The problem discussed in this paper is a knapsack container loading problem with several practical constraints (listed in the Section 1). There are no published approaches that address the container loading problem with these practical constraints. Because it is a knapsack container loading problem, we will briefly discuss some of the recent advances in three-dimensional container loading.

The three-dimensional container loading problem (3D-CLP) can be broadly characterized as type 3/B/O/-, according to the typology presented by Dyckhoff and Finke (1992), or type 3D-R-IIPP/SLOPP/SKP, according to the typology presented by Wäscher et al. (2007). A consignment of goods wrapped up in boxes is assumed to be loaded into a single container of known dimensions, and the boxes and containers are assumed to have a rectangular shape (Junqueira, Morabito, & Sato Yamashita, 2012).

3D-CLP is a typical NP-hard problem (Bischoff & Marriott, 1990) that cannot be solved optimally by an algorithm in polynomial time. When the number of box types increases, exact algorithms are usually confronted with a situation called “combinatorial explosion”. Thus, they can only solve problems with a weak heterogeneous box set. As a result, heuristic methods are usually the first selection for addressing the three-dimensional container loading problem. Heuristic methods may not obtain the best of all of the actual solutions to 3D-CLP, but they can usually produce sufficiently good solutions within acceptable times. Researchers have provided various heuristic methods.

Heuristic methods for 3D-CLP can be divided into two groups according to the method type.

(1) *Tree search methods:* Tree search or graph search methods were successfully utilized in 3D-CLP. Morabito and Arenales (1994) suggested an And/Or graph search method. Eley (2002) tried to fill the container with homogeneous blocks made up of identical items. Hifi (2002) presented a tree search method using hill-climbing strategies. Pisinger (2002) proposed an algorithm that first divides the whole container space into several vertical layers, then divides the layers into a number of horizontal or vertical strips and then generates the strips by solving the one-dimensional knapsack problem. Bortfeldt and Mack (2007) presented a heuristic algorithm that was derived from a branch-and-bound approach. Fanslau and Bortfeldt (2010) proposed an effective tree search algorithm based on the idea of a composite block. Zhang, Peng, and Leung (2012) designed a heuristic block-loading algorithm based on a multi-layer search. Liu, Tan, Xu, and Liu (2014) presented a heuristic wall-building algorithm. Araya and Riff (2014) proposed a beam search approach to the container loading problem.

(2) *Non-tree search methods:* Non-tree search methods include classic heuristic methods and intelligent heuristic methods. The former methods for solving 3D-CLP were presented by Bischoff and Ratcliff (1995), Bischoff, Janetz, and Ratcliff (1995), and Lim, Rodrigues, and Wang (2003), whereas the