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# Stochastic uncoupled dynamics and Nash equilibrium

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#### Abstract

In this paper we consider dynamic processes, in repeated games, that are subject to the natural informational restriction of uncoupledness. We study the almost sure convergence of play (the period-by-period behavior as well as the long-run frequency) to Nash equilibria of the one-shot stage game, and present a number of possibility and impossibility results. Basically, we show that if in addition to random experimentation some recall, or memory, is introduced, then successful search procedures that are uncoupled can be devised. In particular, to get almost sure convergence to pure Nash equilibria when these exist, it suffices to recall the last two periods of play.

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## 1. Introduction

A dynamic process in a multi-player setup is *uncoupled* if the moves of every player do not depend on the payoff (or utility) functions of the other players. This is a natural informational

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requirement, which holds in most models. In Hart and Mas-Colell (2003b) we introduce this concept and show that uncoupled stationary dynamics cannot always converge to Nash equilibria, even if these exist and are unique. The setup was that of deterministic, stationary, continuous-time dynamics.

It is fairly clear that the situation may be different when *stochastic* moves are allowed, since one may then try to carry out some version of exhaustive search: keep randomizing until by pure chance a Nash equilibrium is hit, and then stop there. However, this is not so simple: play has a decentralized character, and no player can, alone, recognize a Nash equilibrium. The purpose of this paper is, precisely, to investigate to what extent Nash equilibria can be reached when considering dynamics that satisfy the restrictions of our previous paper: uncoupledness and stationarity. As we shall see, one can obtain positive results, but these will require that, in addition to the ability to perform stochastic moves of an experimental nature, the players retain some memory from the past plays.

Because we allow random moves, it is easier to place ourselves in a discrete time framework. Thus we consider the repeated play of a given stage game, under the standard assumption that each player observes the play of all players; as for payoffs, each player knows only his own payoff function. We start by studying a natural analog of the approach of our earlier paper; that is, we assume that in determining the random play at time t + 1 the players recall only the information contained in the current play of all players at time t; i.e., past history does not matter. We call this the case of 1-*recall*. We shall then see that the result of our earlier paper is recovered: convergence of play to Nash equilibrium cannot be ensured under the hypotheses of uncoupledness, stationarity, and 1-recall (there is an exception for the case of generic two-player games with at least one pure Nash equilibrium).

Yet, the exhaustive search intuition can be substantiated if we allow for (uncoupled and stationary) strategies with longer recall. Perhaps surprisingly, to guarantee almost sure convergence of play to pure Nash equilibria when these exist, it suffices to have 2-*recall*: to determine the play at t + 1 the players use the information contained in the plays of all players at periods t and t - 1. In general, when Nash equilibria may be mixed, we show that convergence of the long-run empirical distribution of play to (approximate) equilibria can be guaranteed using longer, but finite, recall. Interestingly, however, this does not suffice to obtain the almost sure convergence of the period-by-period behavior probabilities. As it turns out, we can get this too within the broader context of finite memory (i.e., finite-state automata).

In conclusion, one can view this paper as contributing to the demarcation of the border between those classes of dynamics for which convergence to Nash equilibrium can be obtained and those for which it cannot.

The paper is organized as follows. Section 2 presents the model and defines the relevant concepts. Convergence to pure Nash equilibria is studied in Section 3, and to mixed equilibria, in Section 4 (with a proof relegated to Appendix A). We conclude in Section 5 with some comments and a discussion of the related literature, especially the work of Foster and Young (2003a, 2003b).

### 2. The setting

A basic static (one-shot) game is given in strategic (or normal) form, as follows. There are  $N \ge 2$  players, denoted i = 1, 2, ..., N. Each player *i* has a finite set of actions  $A^i$ ; let  $A := A^1 \times A^2 \times \cdots \times A^N$  be the set of action combinations. The payoff function (or utility function) of player *i* is a real-valued function  $u^i: A \to \mathbb{R}$ . The set of randomized or mixed actions of

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