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Nash equilibrium in a spatial model of coalition bargaining

Norman Schofield*, Robert Parks

Center in Political Economy, Campus Box 1208, Washington University in St. Louis, St. Louis, MO 63130, USA

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Abstract

In the model presented here, n parties choose policy positions in a space Z of dimension at least two. Each party is represented by a ''principal'' whose true policy preferences on Z are unknown to other principals. In the first version of the model the party declarations determine the lottery outcome of coalition negotiation. The coalition risk functions are common knowledge to the parties. We assume these coalition probabilities are inversely proportional to the variance of the declarations of the parties in each coalition. It is shown that with this outcome function and with three parties there exists a stable, pure strategy Nash equilibrium, z^* for certain classes of policy preferences. This Nash equilibrium represents the choice by each party principal of the position of the party leader and thus the policy platform to declare to the electorate. The equilibrium can be explicitly calculated in terms of the preferences of the parties and the scheme of private benefits from coalition membership. In particular, convergence in equilibrium party positions is shown to occur if the principals' preferred policy points are close to colinear. Conversely, divergence in equilibrium party positions occurs if the bliss points are close to symmetric. If private benefits (the non-policy perquisites from coalition membership) are sufficiently large (that is, of the order of policy benefits), then the variance in equilibrium party positions is less than the variance in ideal points. The general model attempts to incorporate party beliefs concerning electoral responses to party declarations. Because of the continuity properties imposed on both the coalition and electoral risk functions, there will exist mixed strategy Nash equilibria. We suggest that the existence of stable, pure strategy Nash equilibria in general political games of this type accounts for the non-convergence of party platforms in multiparty electoral systems based on proportional representation. © 2000 Elsevier Science B.V. All rights reserved.

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*Corresponding author. Tel.: +1-314-935-4774; fax: +1-314-935-4156.
E-mail address: schofld@wueconb.wustl.edu (N. Schofield)

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1. Introduction

Spatial models of competitive party behavior in representative democracies often give ‘‘convergence’’ results that are at odds with the way political parties seem to behave. In two-party competition, for example, it is usually assumed that the only motivation of a party is to win as many seats as possible. If the electoral model is ‘‘deterministic’’, then pure strategy Nash equilibria will generally not exist (Saari, 1996), but the support of the mixed strategy Nash equilibria will be within the so-called ‘‘uncovered set’’ (Banks et al., 1998; Cox, 1987; McKelvey, 1986). This set is centrally located with respect to the distribution of voter ideal points. If the electoral model is probabilistic, or ‘‘stochastic’’, and parties choose positions to maximize expected vote, then the pure strategy Nash equilibrium will also be centrally located, at the mean of the voter distribution (Coughlin, 1992; Enelow and Hinich, 1984; Hinich, 1977).

A two-party model, due to Cox (1984), does not exhibit such ‘‘Downsian’’ (Downs, 1957) convergence, because it assumes that elections are inherently risky and that parties actually care about policy. Cox supposes that each party, i, has a ‘‘sincere’’ point, o_i, say, in the policy space, Z, and that the electoral outcome of a pair of party declarations, $z=(z_1, z_2)$, is described by a ‘‘lottery’’ $\{(p_1(z), z_1), (p_2(z), z_2), (p_o(z), z_o)\}$. Here $p_i(z)$ for $i=1, 2$, is the probability that party i wins, and implements z_i, while $p_o(z)$ is the probability of a draw after which $\{1, 2\}$ form a government and implement a compromise position, z_o, say. Given ideal points (o_1, o_2) for the two parties, and appropriate ‘‘spatially’’ defined utilities, Cox suggests there is a pure strategy Nash equilibrium $(z_1^*, z_2^*) \in Z^2$. One problem not fully addressed by Cox is why a winning party i say, would choose to implement z_i^* after the election, rather than its preferred position, o_i. (See Banks, 1990, for a discussion.) One way to deal with this problem is to suppose that each party, i, is a heterogeneous collection of elite actors, described by $\{o_{k_i}\}$ who choose one of their members as the party principal. This principal then chooses a second elite member as leader, or agent for the party, whose ideal point is identical to the party declaration, z_i. By this method, Nash equilibrium selection of leaders gives policy choices which are credible to the electorate. However, Cox's model of competition does not readily generalize to the multiparty case (where the number of parties, n, is at least three). It is typical of such a situation that no party wins a majority, and that coalitions are necessary for government formation (Laver and Schofield, 1990).

Recent analyses of multiparty competition, based on stochastic models of electoral behavior and expected vote maximizing behavior, have extended the earlier two-party analysis of Downs (1957) and have shown that the unique Nash equilibria are ‘‘convergent’’ and at the mean of the electoral distribution (Lin et al., forthcoming). In contrast, if the voter model is deterministic, then typically there are no pure strategy Nash equilibria. Indeed, it is possible that discontinuity in the electoral response means that there are not even mixed strategy equilibria (Dasgupta and Maskin, 1986; Osborne, 1993).

However, the empirical evidence available (Budge, 1987; Laver and Schofield, 1990; Schofield, 1995a) suggests that the selection of party positions, under electoral systems based on proportional representation, is neither convergent (as suggested by the stochastic model), nor chaotic (as indicated by the deterministic model).