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Profit sharing in unique Nash equilibrium: Characterization in the two-agent case

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Abstract

Two agents jointly operate a decreasing marginal returns technology to produce a private good. We characterize the class of output-sharing rules for which the labor-supply game has a unique Nash equilibrium. It consists of two families: rules of the *serial* type which protect a small user from the negative externality imposed by a large user, and rules of the *reverse serial* type, where one agent effectively employs the other agent's labor. Exactly two rules satisfy symmetry; a result in sharp contrast with Moulin and Shenker's characterization of their serial mechanism as the unique *cost*-sharing rule satisfying the same incentives property [Moulin, H., Shenker, S., 1992. Serial cost sharing. Econometrica 60 (5), 1009–1037]. We also show that the familiar *stand-alone test* characterizes the class of *fixed-path methods* under our incentives criterion [Friedman, E.J., 2004. Strong monotonicity in surplus sharing. Econ. Theory 23, 643–658].

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1. Introduction

When several producers jointly operate a production process, total output (or profits) must be shared as a function of their individual contributions (see Israelsen, 1980; Sen, 1966; Weitzman, 1974). This question applies whether the production structure is one of common access to the production function (as in the so-called "commons problem") or one where property rights to the

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technology are clearly defined. An extreme example of the latter is that of a monopolist hiring workers.

The production possibilities are common knowledge and exhibit decreasing marginal returns, but the individual leisure-consumption trade-offs are private information. We are concerned with sharing rules with very strong incentives properties, so as to avoid undesirable phenomena like free-riding or the familiar "tragedy of the commons." We demand that the input supply game induced by the sharing rule, in which agents decide how much input to contribute to the production process, admits a unique Nash equilibrium at all preference profiles. We refer to this incentives criterion as *unique Nash incentive compatibility* (*UNIC*). This criterion is in fact stronger than that of *strategyproofness* (*SP*), under which it is a dominant strategy for every agent to behave according to her true preference. Like SP, UNIC does not hinge on any informational assumptions and is therefore more robust than, say, Bayesian incentive compatibility.[1]

In the two-agent case, we characterize the class of sharing rules which are monotonic (each agent's share is increasing in her own input contribution), smooth (the sharing rule is continuously differentiable in inputs) and satisfy UNIC (Theorem 1). This class of sharing rules is made up of two families which we call the "serial" family and the "reverse serial" family. An essential feature of rules in the serial family is that the share of a relatively small supplier of input is unaffected by changes in the supply level of a large supplier (a feature called the "serial principle" in Sprumont, 1998) while the converse is true for rules of the reverse serial type: the share of a large supplier is unaffected by changes in the input level of a small supplier.

In addition, the externality imposed by a small user on large users is typically negative under a serial rule. Conversely, a large supplier of input typically imposes positive externalities on small suppliers under a reverse serial rule. Thus, with the exception of priority rules (which belong to both families), we argue that serial rules are more adapted to the commons problem, with the negative externality reflecting congestion, while reverse serial rules correspond to a more "corporate" production structure where the owner of the facility extracts rents from the labor contribution of a worker.

In Section 5 we consider a popular axiom in the commons literature. The *stand-alone test* (*SA*) captures the essence of the commons problem by demanding that no agent be made better off by the presence of others than if she were operating the technology by herself (see, e.g., Moulin and Shenker, 1992; Suh, 1997; Sprumont, 1998; Hougaard and Thorlund-Petersen, 2000). It turns out that SA characterizes the output-sharing version of the class of *fixed-path methods* (*FPMs*) discussed in Friedman (2002, 2004) (see Theorem 2). These sharing rules allocate marginal quantities of input, and the corresponding amounts of output, along a prespecified path in the agents' input space. Among well-known FPMs are the Moulin and Shenker serial rule and priority rules, which follow the diagonal of the positive orthant and an axis of the agents' input space, respectively.

2. Relation to the literature

This work contributes to the large literature on the trade-off between efficiency and incentive compatibility.

Because mechanisms in the serial and reverse serial family satisfy UNIC, they fail to be first-best efficient (see Leroux, 2004). In fact, when coupled with the requirement that agents be

[1] We refer the reader to a companion paper (Leroux, 2007) for a discussion of strategyproofness and UNIC relative to Bayesian incentive compatibility.