



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Journal of Mathematical Economics 41 (2005) 351–385

JOURNAL OF
Mathematical
ECONOMICS

www.elsevier.com/locate/jmateco

Nash equilibria in a model of multiproduct price competition: an assignment problem

Ivan Arribas, Amparo Urbano*

Departamento de Análisis Económico, Universidad de Valencia, Campus dels Tarongers, Edificio Departamental Oriental, Avda. dels Tarongers, s/n, 46022 Valencia, Spain

Received 5 February 2003; received in revised form 16 October 2003; accepted 4 December 2003

Available online 20 February 2004

Abstract

We study the market interaction of a finite number of single-product firms and a representative buyer, where the buyer consumes bundles of these goods. The buyers' value function determines their willingness to pay for subsets of goods. We show that Nash-equilibrium outcomes are solutions of the linear relaxation of an integer programming assignment problem and that they always exist. The Nash-equilibrium price set is characterized by the Pareto frontier of the associated dual problem's projection on the firms' price vectors. We identify the Nash-equilibrium prices for monotonic buyers' value functions and, more importantly, we show that some central solution concepts in cooperative game theory are (subgame perfect) equilibrium prices of our strategic game.

© 2004 Elsevier B.V. All rights reserved.

JEL classification: C72; D21; D41; D43; L13

Keywords: Multiproduct price competition; Integer programming; Subgame perfect nash equilibria

1. Introduction

We study the market interaction of a finite number of firms and a representative buyer, where each firm produces at most one good and the buyer consumes bundles of these goods. We show that Nash equilibrium outcomes are solutions of the linear relaxation of an integer programming assignment problem, where subsets of goods are assigned to the buyer.

Strategic interaction in markets has been thoroughly studied in the literature, and has mostly focused on the case of several firms which produce a homogeneous good or goods

* Corresponding author. Tel.: +34-96-382-82-07; fax: +34-96-382-82-49.

E-mail addresses: ivan.arribas@uv.es (I. Arribas), amparo.urban@uv.es (A. Urbano).

which are close substitutes. The most well known works are the Cournot quantity and the Bertrand price competition models. However, the scenario where firms produce goods of a very general nature has received little attention.

The literature on strategic multiproduct oligopolistic competition deals with product differentiation, where each firm produces a single good with different characteristics, address models. Here we find models of spatial competition, where firms differ in their location (see Hotelling, 1929; d'Aspremont et al., 1979; Salop, 1979, among others) and models of quality differentiation where a number of firms produce substitute goods that differ in quality and then sell to a continuum of buyers, identical in tastes but different in income (e.g. Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983). In address models, firms try to relax competition through product differentiation either by increasing distance from rival firms or by considering quality gaps. On the other hand, the existence of pure strategy equilibrium in a standard multiproduct Bertrand competition, has been shown by Milgrom and Roberts (1990), Milgrom and Shannon (1994), under some restrictions on the demand and cost functions, and when all goods are only either substitutes or complements.

A different approach is that of Tauman et al. (1997, TUW, hereafter) who study a simple model of price competition in a multiproduct oligopoly market where goods can be of a very general nature and not necessarily substitutes or complements. Firms produce only one good and buyers are all identical. The (representative) buyer purchases a bundle of goods and either one or zero units of each of the products are chosen. Moreover, she is characterized by her willingness to pay for every subset of goods. Firms are engaged in price competition in the first stage and buyers take their consumption decisions in the second stage. They show that a pure strategy equilibrium always exists and that the equilibrium consumption is always efficient in the sense of maximizing total social surplus. The equilibrium prices depend on the buyer's value function, i.e. the buyer's willingness to pay as a function of the consumption bundle. However, the price function can only be characterized either under concavity or convexity of the buyer's value function.

Our work generalizes the analysis of TUW (1997) and characterizes the set of all Nash-equilibrium outcomes. Since the demand function is unitary and buyers purchase bundles of goods, the equilibrium analysis leads us towards combinatorial complexity which makes it necessary to use integer linear programming to better handle the model's indivisibilities. Specifically, we offer the set of equilibrium prices as some solutions of the associated dual to the linear relaxation of an integer programming assignment problem.

We show that the solutions of this linear programming problem are the Nash-equilibrium prices and consumption set. It is interesting to note that the solutions of a linear programming problem are a convex polyhedron, and so is the projection of the dual problem's solutions on firms' price vectors. This polyhedron is completely determined by its vertices. The Pareto frontier of the above projection has to be identified in order to characterize the set of *all* Nash-equilibrium price vectors. As this frontier can be expressed as the convex combination of non-Pareto dominated vertices, we just need to obtain all these vertices. At every Nash equilibrium, a non-active firm may set prices equal to or different from marginal costs, and the selected firms' prices are the non-Pareto dominated vectors of the above Pareto frontier. The equilibrium consumption set is always efficient. Moreover, equilibrium prices belong to the core of the economy and, since in this model the core is

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات