Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of Economic Theory 147 (2012) 2357–2375

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Implementation in mixed Nash equilibrium ☆

Claudio Mezzetti [a,*], Ludovic Renou [b]

[a] *Department of Economics, University of Melbourne, Parkville, VIC 3010, Australia*
[b] *Department of Economics, University of Essex, Colchester CO4 3SQ, United Kingdom*

Received 14 June 2010; final version received 18 December 2011; accepted 18 April 2012

Available online 1 October 2012

Abstract

A mechanism implements a social choice correspondence f in mixed Nash equilibrium if, at any preference profile, the set of *all* (pure and mixed) Nash equilibrium outcomes coincides with the set of f-optimal alternatives for all cardinal representations of the preference profile. Unlike Maskin's definition, our definition does not require each optimal alternative to be the outcome of a *pure* equilibrium. We show that set-monotonicity, a weakening of Maskin's monotonicity, is necessary for mixed Nash implementation. With at least three players, set-monotonicity and no-veto power are sufficient. Important correspondences that are not Maskin monotonic can be implemented in mixed Nash equilibrium.

JEL classification: C72; D71

Keywords: Implementation; Maskin monotonicity; Pure and mixed Nash equilibrium; Set-monotonicity; Social choice correspondence

1. Introduction

This paper studies the problem of implementation in mixed Nash equilibrium. According to our definition, a mechanism implements an ordinal social choice correspondence f in mixed Nash equilibrium if, at any preference profile, the set of all (pure and mixed) equilibrium outcomes corresponds to the set of f-optimal alternatives for all cardinal representations of the preference profile. Crucially, and unlike the classical definition of implementation, this definition of implementation does not give a predominant role to pure equilibria: an f-optimal alternative

☆ We thank Olivier Tercieux, three referees and the associate editor for insightful comments on the paper.
* Corresponding author.
E-mail addresses: cmez@unimelb.edu.au (C. Mezzetti), lrenou@essex.ac.uk (L. Renou).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2012.09.004

does not have to be the outcome of a pure Nash equilibrium. At the same time, we maintain an entirely ordinal approach. We assume that a social choice correspondence f maps profiles of preference orderings over alternatives into subsets of alternatives (not lotteries) and we require that a given mechanism implements f irrespective of which cardinal representation is chosen.

Most of the existing literature on Nash implementation does not consider equilibria in mixed strategies (see Jackson [16] and Maskin and Sjöström [18], for excellent surveys).[1] Perhaps, the emphasis on pure equilibria expresses a discomfort with the classical view of mixing as deliberate randomizations on the part of players. However, it is now accepted that even if players do not randomize but choose definite actions, a mixed strategy may be viewed as a representation of the other players' uncertainty about a player's choice (e.g., see Aumann and Brandenburger [3]). Moreover, almost all mixed equilibria can be viewed as pure Bayesian equilibria of nearby games of incomplete information, in which players are uncertain about the exact profile of preferences, as first suggested in the seminal work of Harsanyi [14]. This view acknowledges that games with commonly known preferences are an idealization, a limit of near-complete information games. This interpretation is particularly important for the theory of implementation in Nash equilibrium, whereby the assumption of common knowledge of preferences, especially on large domains, is at best a simplifying assumption.

Furthermore, recent evidence in the experimental literature suggests that equilibria in mixed strategies are good predictors of behavior in some classes of games e.g., coordination games and chicken games (see Chapters 3 and 7 of Camerer [10]). Since, for some preference profiles, a mechanism can induce one of those games, paying attention to mixed equilibria is important if we want to describe or predict players' behavior. While we find no compelling reasons to give pure Nash equilibria a special status, we follow an ordinal approach because we believe it imposes a welcome degree of robustness on society's preferences and the mechanism adopted. Thus, we require that the set of f-optimal outcomes only depends on players' ordinal preferences, and that the mechanism adopted implements f for all possible cardinal representations of those ordinal preferences.

Our definition of mixed Nash implementation yields novel insights. We demonstrate that the condition of Maskin monotonicity is not necessary for full implementation in mixed Nash equilibrium. Intuitively, consider a profile of preferences and an alternative, say a, that is f-optimal at that profile of preferences. According to Maskin's definition of implementation, there must exist a pure Nash equilibrium with equilibrium outcome a. Thus, any alternative a player can obtain by unilateral deviations must be less preferred than a. Now, if we move to another profile of preferences where a does not fall down in the players' ranking, then a remains an equilibrium outcome and must be f-optimal at that new profile of preferences. This is the intuition behind the necessity of Maskin monotonicity for Nash implementation. Unlike Maskin's definition of implementation, our definition does not require a to be a pure equilibrium outcome. So, suppose that there exists a mixed equilibrium with a as an equilibrium outcome.[2] The key observation

[1] Two notable exceptions are Maskin [17] for Nash implementation and Serrano and Vohra [23] for Bayesian implementation. These authors do consider mixed equilibria, but still require each f-optimal alternative to be the outcome of a *pure* equilibrium; pure equilibria are given a special status. While Maskin [17] shows that eliminating unwanted mixed strategy equilibria imposes no additional restriction to Nash implementation, Serrano and Vohra [23] show that significantly more restrictive conditions than in Jackson [15] are required to implement social choice correspondences in Bayesian equilibrium.

[2] More precisely, let σ^* be the mixed Nash equilibrium and $\mathbb{P}_{\sigma^*,g}$ the distribution over alternatives induced by the strategy profile σ^* and the allocation rule g. Then a belongs to the support of $\mathbb{P}_{\sigma^*,g}$.