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A class of interference induced games: Asymptotic Nash equilibria and parameterized cooperative solutions[☆]

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ABSTRACT

We consider a multi-agent system with linear stochastic individual dynamics, and individual linear quadratic ergodic cost functions. The agents partially observe their own states. Their cost functions and initial statistics are a priori independent but they are coupled through an interference term (the mean of all agent states), entering each of their individual measurement equations. While in general for a finite number of agents, the resulting optimal control law may be a non linear function of the available observations, we establish that for certain classes of cost and dynamic parameters, optimal separated control laws obtained by ignoring the interference coupling, are asymptotically optimal when the number of agents goes to infinity, thus forming for finite N, an ϵ-Nash equilibrium. More generally though, optimal separated control laws may not be asymptotically optimal, and can in fact result in unstable overall behavior. Thus we consider a class of parameterized decentralized control laws whereby the separated Kalman gain is treated as the arbitrary gain of a Luenberger like observer. System stability regions are characterized and the nature of optimal cooperative control policies within the considered class is explored. Numerical results and an application example for wireless communications are reported.

1. Introduction

There has been a surge of interest in the study and analysis of large population stochastic multi-agent systems due to their wide variety of applications over the past several years. Many practical applications and examples of these systems arise in engineering, biological, social and economic fields, such as wireless sensor networks (Chong & Kumar, 2003), very large scale robotics (Reif & Wang, 1999), controlled charging of a large population of electric vehicles (Karfopoulos & Hatziargyriou, 2013), synchronization of coupled oscillators (Yin, Mehta, Meyn, & Shanbhag, 2012), swarm and flocking phenomenon in biological systems (Grönbaum & Okubo, 1994; Passino, 2002), evacuation of large crowds in emergency situations (Helbing, Farkas, & Vicsek, 2000; Lachapelle, 2010), sharing and competing for resources on the Internet (Altman et al., 2006), to cite a few. Large-scale stochastic games with unbounded costs were studied in Adlakha et al. (2008). Mean field game theory, which addresses a class of dynamic games with a large number of agents in which each agent interacts with the average or so-called mean field effect of other agents via couplings in their individual dynamics and cost functions, was studied in Huang, Caines and Malhamé (2006), Huang, Caines, and Malhamé (2012), Lachapelle and Lions (2007), Nourian, Caines, Malhamé, and Huang (2012), Nourian, Caines, Malhamé, and Huang (2013), Wang and Zhang (2012) and Wang and Zhang (2014). In Li and Zhang (2008), the mean field linear quadratic Gaussian (LQG) framework was extended to systems of agents with Long Time Average (LTA) (i.e., ergodic) cost functions such that the set of control laws possesses an almost sure (a.s.) asymptotic Nash equilibrium property.

Stochastic Nash games with partial observation have been of interest since the late 1960s. LQG continuous-time zero-sum stochastic games with output measurements corrupted by additive independent white Gaussian noise were studied in Rhodes and Luenberger (1969a,b) under the constraint that each player is limited to a linear state estimator for generating its optimal controls. These results were extended to nonzero-sum Nash

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games in Saksena and Cruz (2005). In these works the authors assumed that the separation principle holds. In Kian, Cruz, and Simaan (2002), discrete-time nonzero-sum LQG Nash games with constrained state estimators and two different information structures were investigated, where it is shown that the optimal control laws do not satisfy the separation principle and the estimator characteristics depend on the controller gains.

Distributed decision-making with partial observation for large population stochastic multi-agent systems was studied in Caines and Kizilkale (2013, 2014); Huang, Caines and Malhamé (2006); Wang and Zhang (2013), where the synthesis of Nash strategies is investigated for the agents that are weakly coupled through either individual dynamics or costs. In Abedinpour Fallah et al. (2013a); Abedinpour Fallah Malhamé and Martinelli (2013b); Abedinpour Fallah, Malhamé and Martinelli (2014) the authors studied a somewhat dual situation whereby large populations of partially observed stochastic agents, although a priori individually independent, are coupled only via their observation structure. The latter involves an interference term depending on the empirical mean of all agent states. The study of such measurement-coupled systems is inspired by a variety of applications, including for instance the communications model for power control in cellular telephone systems (Huang, Caines, & Malhamé, 2004; Perreau & Anderson, 2006), where any conversation in a cell acts as interference on the other conversations in that cell. Indeed, despite the so-called signal processing gain achieved thanks to a user's specific coding advantage (and considered in our model to be of order $1/N$ where N is the total number of agents), the ability of the base station to correctly decode the signals sent by a given mobile, remains limited by interference formed by the superposition of all other in cell user signals. Viewed in this light, the studied problem can be considered as a game over a noisy channel.

Individual agent dynamics are assumed to be linear, stochastic, with linear local state measurements, and in the current paper, we focus on the case where the measurements interaction model is assumed to depend only on the empirical mean of agents states in a purely additive manner. In general, in such decentralized control problems, the measurement system could be used for some sort of signaling, and control and estimation are typically coupled (Witsenhausen, 1968). We assume that each agent is constrained to use a linear Kalman filter-like state estimator to generate its optimal strategies. For a finite number of agents, we establish that for certain classes of cost and dynamic parameters, optimal separated control laws obtained by ignoring the interference coupling, are asymptotically optimal when the number of agents goes to infinity, thus forming for finite N, an ϵ-Nash equilibrium. More generally though, optimal separated control laws may not be asymptotically optimal, and can in fact result in unstable overall behavior. Thus we consider a class of parameterized decentralized control laws whereby the separated Kalman gain is treated as the arbitrary gain of a Luenberger like observer. System stability regions are characterized and the nature of optimal cooperative control policies within the considered class is explored.

The rest of the paper is organized as follows. The problem is defined and formulated in Section 2. Section 3 presents the closed-loop dynamics model. In Section 4, a decentralized control and state estimation algorithm via stability analysis is described and a characterization of its optimality properties is given. Section 5 presents parameterized cooperative solutions. Also, both Sections 4 and 5 provide some numerical simulation results. Section 6 presents an application example for wireless communications. Concluding remarks are stated in Section 7.

2. Problem formulation

Consider a system of N agents, with individual scalar dynamics for simplicity of computations. The evolution of the state component is described by

$$x_{k+1,i} = ax_{k,i} + bu_{k,i} + w_{k,i} \tag{1}$$

with partial scalar state observations given by:

$$y_{k,i} = cx_{k,i} + h\left(\frac{1}{N}\sum_{j=1}^{N} x_{k,j}\right) + v_{k,i} \tag{2}$$

for $k \geq 0$ and $1 \leq i \leq N$, where $x_{k,i}, u_{k,i}, y_{k,i} \in \mathbb{R}$ are the state, the control input and the measured output of the ith agent, respectively. The random variables $w_{k,i} \sim \mathcal{N}(0, \sigma_w^2)$ and $v_{k,i} \sim \mathcal{N}(0, \sigma_v^2)$ represent independent Gaussian white noises at different times k and at different agents i. The Gaussian initial conditions $x_{0,i} \sim \mathcal{N}(\bar{x}_0, \sigma_0^2)$ are mutually independent and are also independent of $\left\{w_{k,i}, v_{k,i}, 1 \leq i \leq N, k \geq 0\right\}$. σ_w^2, σ_v^2 and σ_0^2 denote the variance of $w_{k,i}$, $v_{k,i}$ and $x_{0,i}$, respectively. Moreover, a is a scalar parameter and b, c, $h > 0$ are positive scalar parameters.

The problem to be considered is to synthesize the linear time invariant decentralized separated policies such that each agent is stabilized by a feedback control of the form

$$u_{k,i} = -f\hat{x}_{k,i}, \tag{3}$$

where $\hat{x}_{k,i}$ is an estimator of $x_{k,i}$ based only on local observations of the ith agent, and f is a constant scalar gain. For the purposes of the paper, the class of *decentralized separated policies* (3) includes all control policies satisfying the following three conditions: (i) they are defined by two time invariant feedback gains K and f, (ii) they are separated in that the control is a linear feedback $-f\hat{x}_{k,i}$ on the state estimate of $x_{k,i}$, while the state estimate $\hat{x}_{k,i}$ is obtained from a Luenberger like observer equation under the assumed state estimate feedback structure, i.e., it evolves according to:

$$\hat{x}_{k+1,i} = (a - bf)\hat{x}_{k,i} + K(y_{k+1,i} - c(a - bf)\hat{x}_{k,i}), \tag{4}$$

(iii) they are decentralized in that the state estimate is based solely on agent based observations $y_{k,i}$.

Furthermore, when the gain K is the Kalman gain as obtained when assuming zero interference in the local measurements (setting $h = 0$ in (2)), the resulting estimator (4) will be called the *naive* Kalman filter. Moreover, the individual cost function for each agent is given by

$$J_i \triangleq \overline{\lim_{T\to\infty}} \frac{1}{T}\mathbb{E}\sum_{k=0}^{T-1}(x_{k,i}^2 + ru_{k,i}^2) \tag{5}$$

where $r > 0$ is a positive scalar parameter.

Assumption 1. To simplify the synthesis procedure we assume zero mean for initial conditions of all agents, i.e., $\mathbb{E}x_{0,i} = \bar{x}_0 = 0, i \geq 1$.

Remark 1. To show that the decentralized control problem formulated here is a game, let us assume for the sake of discussion that the original agent dynamics is unstable. Then it suffices to observe that, for finite N at least, the inability of a single agent to stabilize its own dynamics would have direct consequences on the ability of other agents to stabilize their own, hence demonstrating the impact of that agent on other agents' individual costs.

3. Closed-loop dynamics model

3.1. Closed-loop agent dynamics

In this section first we obtain the 4th order model of the closed-loop agent dynamics. In particular, when local state estimate