



Neural network for solving Nash equilibrium problem in application of multiuser power control



Xing He^{a,*}, Junzhi Yu^b, Tingwen Huang^c, Chuandong Li^a, Chaojie Li^d

^a School of Electronics and Information Engineering, Southwest University, Chongqing 400715, PR China

^b State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, PR China

^c Department of Mathematics, Texas A&M University at Qatar, Doha, P.O. Box 23874, Qatar

^d School of Electrical and Computer Engineering, RMIT University, Melbourne, VIC 3001, Australia

ARTICLE INFO

Article history:

Received 30 March 2014
Received in revised form 22 May 2014
Accepted 1 June 2014
Available online 6 June 2014

Keywords:

Neural network
Multiuser power control
Nash game
Global convergence

ABSTRACT

In this paper, based on an equivalent mixed linear complementarity problem, we propose a neural network to solve multiuser power control optimization problems (MPCOP), which is modeled as the noncooperative Nash game in modern digital subscriber line (DSL). If the channel crosstalk coefficients matrix is positive semidefinite, it is shown that the proposed neural network is stable in the sense of Lyapunov and global convergence to a Nash equilibrium, and the Nash equilibrium is unique if the channel crosstalk coefficients matrix is positive definite. Finally, simulation results on two numerical examples show the effectiveness and performance of the proposed neural network.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

Due to low infrastructure cost and high speed data communication, digital subscriber line (DSL) technology has become a widely-used method for broadband internet access. In the DSL, power control of the design of interference-limited multiuser communication systems is a central issue, and it also has attracted significant attention from both academia and industry. Power control refers to forcing each user to transmit enough power so that it can achieve the required quality without causing unnecessary interference to other users in these systems. Therefore, the system design involves a performance tradeoff among the different users.

In the DSL, multiuser power control is an optimization problem, and a typical measure of system throughput is the sum of all users' rates (Cendrillon, Moonen, Verliden, Bostoen, & Yu, 2004; Cherubini, Eleftheriou, & Olcer, 2000; Song, Chung, Ginis, & Cioffi, 2002). Unfortunately, in this framework, the optimization problem of maximizing the sum rates is nonconvex with many local solutions (Song et al., 2002). In Cherubini et al. (2000), Cherubini et al. proposed a simulated annealing method to obtain a global optimal power allocation solution, which suffers from slow convergence and lack a rigorous analysis. Another method which has been

very successful in solving multiuser power control is the game theoretic approach. Yu, Ginis, and Cioffi (2002) considered the multiuser power control problem in a frequency-selective interference channel, which is modeled as a noncooperative game, and its key observation is that each DSL user's data rate is a concave function of its own power spectra vector when the interfering users' power vectors are fixed. Luo and Pang (2006) presented a convergence analysis of iterative water-filling algorithm in more realistic channel settings and for arbitrary number of users. In Yamashita and Luo (2008), multiuser power control problem was formulated to find a Nash equilibrium of the DSL game as a nonlinear complementarity problem. In Cendrillon, Yu, Moonen, Verlinden, and Bostoen (2010), Cendrillon et al. discussed a centralized algorithm for optimal spectrum balancing. In Pang, Scutari, Facchinei, and Wang (2008), Pang et al. introduced the minimization of transmit power in Gaussian parallel interference channels subject to a rate constraint for each user. Therefore, many researchers have made deep research into the algorithm of multiuser power control optimization problem in the DSL.

In the application of multiuser power control optimization problems (MPCOP), real-time solutions are often needed. However, classical optimization methods are not competent for problems with high dimensionality or stringent computation time requirement. In the past two decades, the essence of neural network optimization lies in its inherent nature of parallel and distributed information processing and the availability of hardware implementation (Hopfield & Tank, 1985; Tank & Hopfield, 1986).

* Corresponding author. Tel.: +86 15215156680.

E-mail addresses: hexingdoc@swu.edu.cn (X. He), junzhi.yu@ia.ac.cn (J. Yu), tingwen.huang@qatar.tamu.edu (T. Huang), licd@cqu.edu.cn (Chuandong Li), cjlee.cqu@163.com (Chaojie Li).

Since the seminal work of Hopfield and Tank, there has been increasing interest in investigating the theory, methodology and applications of recurrent neural networks for optimization. Based on penalty functions, Lagrange functions and primal and dual functions, many neural networks for solving various optimization problems (Bian & Chen, 2012; Cheng, Hou, & Tan, 2008; Forti, Nistri, & Quincampoix, 2006; Gao, Liao, & Qi, 2005; He, Li, Huang, & Li, 2014a, 2014b; Hosseini, Wang, & Mohamma, 2013; Hu & Wang, 2006, 2007; Hu & Zhang, 2009; Liu, Cao, & Chen, 2010; Liu, Guo, & Wang, 2012; Liu & Wang, 2011, 2013; Wang, 1993; Xia, 1996; Xia, Leung, & Wang, 2002; Xia & Wang, 2004) were developed, and these neural networks improved performance in terms of global convergence and parallel computational implementability.

In this paper, motivated by the effectiveness and efficiency of neural network optimization method, we have attempted to solve MPCOP using neural network approach. Compared with many iterative algorithms, our contribution is to design neural network for solving multiuser power control problem. Following the method (Luo & Pang, 2006), the Karush–Kuhn–Tucker optimality conditions of the noncooperative Nash game which results from MPCOP are reformulated as equivalent mixed linear complementarity problem (LCP). Based on LCP, a projection neural network is proposed for solving this Nash game. Using Lyapunov function theory, it is shown that the proposed neural network is Lyapunov stable and globally convergent to Nash equilibria sets, and under arbitrary symmetric interference environment and certain asymmetric channel condition, it is proven that the proposed neural network is globally convergent to unique Nash equilibrium. Finally, simulation results on numerical examples show the effectiveness and performance of the neural network for solving MPCOP.

The remainder of this paper is organized as follows. In the next section, the noncooperative Nash game about MPCOP is described, and a projection neural network is proposed to solve the noncooperative Nash game. The convergence of the proposed neural network is proved in Section 3. In Section 4, simulation results on two numerical examples are given. Finally, Section 5 concludes this paper.

Notation: Given column vectors $x = (x_1, x_2, \dots, x_n)^T$, $(x)^+ = ((x_1)^+, (x_2)^+, \dots, (x_n)^+)^T$, $(x_i)^+ = \max(0, x_i)$, $1_n = \underbrace{[1, 1, \dots, 1]}_n$, $x > 0$ means all the $x_i > 0$. $\|x\|$ denotes l_2 norm. $\forall a, b \in R^1$, $a \perp b$ denotes $a \cdot b = 0$. For $K \subset R^n$, the projection operator $H_K(x)$ is defined by: $H_K(x) = \arg \min_{y \in K} \|x - y\|$.

2. Problem formulation and model description

In this section, we introduce the game theoretic formulation of MPCOP in the DSL model. Then we will construct a recurrent neural network for solving MPCOP. Consider a tuple (m, \mathcal{F}, Ω) where m is the number of users in digital subscriber lines, n denotes the total number of frequency tones available to the DSL users. $\mathcal{F} = \{(f_i)_{i=1}^m\}$ is the set of user-specific objective functions, each user controls the variable $p^i = (p_1^i, p_2^i, \dots, p_n^i)^T \in R^n$, which denotes the power spectra vector of user i with p_k^i signifying the power allocated to frequency tone k , and we denote by p the overall vector of all variables $p = ((p^1)^T, (p^2)^T, \dots, (p^m)^T)^T \in R^{mn}$, and the user-specific strategy sets Ω are denoted by $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_m\}$, $\Omega_i \subseteq R^n$, and Ω_i is described by

$$\Omega_i = \left\{ p^i \in R^n \mid 0 \leq p_k^i \leq CAP_k^i, \forall k = 1, \dots, n, \sum_{k=1}^n p_k^i \leq P_{\max}^i \right\},$$

where CAP_k^i and P_{\max}^i are some positive constants. Taking p_k^i for $j \neq i$ as fixed, user i solve the following concave maximization

problem

$$\max_{p^i} f_i(p^i; p^{-i}) \equiv \sum_{k=1}^n \log \left(1 + \frac{p_k^i}{\sigma_k^i + \sum_{j \neq i} \alpha_k^{ij} p_k^j} \right) \quad \text{s.t. } p^i \in \Omega_i, \quad (1)$$

where $p = (p^1; p^{-1})$, p^i is the strategy of player i and p^{-i} are the strategies of all the users except i , σ_k^i are positive scalars and α_k^{ij} are nonnegative scalars for all $i \neq j$, and all k representing noise power spectra and channel crosstalk coefficients, respectively.

Then a Nash equilibrium is a feasible $p^* = ((p^{1,*})^T, (p^{2,*})^T, \dots, (p^{m,*})^T)^T$ such that $f_i(p^{i,*}; p^{-i,*}) \geq f_i(y^i; p^{-i,*})$, $\forall y^i \in \Omega_i$, $\forall i = 1, 2, \dots, m$. In the following, we state some assumptions about the cost functions and strategy sets.

Assumption 1. In the feasible set Ω_i , $P_{\max}^i < \sum_{k=1}^n CAP_k^i$.

Assumption 2. $\alpha_k^{ii} = 1$ for all k and i .

For problem (1), define a Lagrange function

$$L_i(p^i, u^i, \gamma^i) = -f_i + u^i \left(\sum_{k=1}^n p_k^i - P_{\max}^i \right) + \sum_{k=1}^n \gamma_k^i (p_k^i - CAP_k^i), \quad (2)$$

where $u^i \in R^1$, $\gamma_k^i \in R^1$ are the multiplier of the inequality $\sum_{k=1}^n p_k^i \leq P_{\max}^i$, $p_k^i \leq CAP_k^i$, respectively. According to the well-known saddle point theorem (Body & Vandenberghe, 2003), we can get the KKT conditions for user i of problem (1) as follows:

$$\begin{aligned} 0 &\leq p_k^i \perp -\frac{1}{\sigma_k^i + \sum_{i=1}^m \alpha_k^{ij} p_k^j} + u_i + \gamma_k^i \geq 0, \quad \forall k = 1, 2, \dots, n \\ 0 &\leq u^i \perp P_{\max}^i - \sum_{k=1}^n p_k^i \geq 0 \\ 0 &\leq \gamma_k^i \perp CAP_k^i - p_k^i \geq 0, \quad \forall k = 1, 2, \dots, n \end{aligned} \quad (3)$$

p^* is a solution of problem (1) if and only if there exist $u^* = (u^{1,*}, u^{2,*}, \dots, u^{m,*})^T \in R^m$ and $\gamma^* = ((\gamma^{1,*})^T, (\gamma^{2,*})^T, \dots, (\gamma^{m,*})^T)^T \in R^{mn}$ such that (p^*, u^*, γ^*) satisfies system (3). From Luo and Pang (2006), we have the following results.

Lemma 1. Under Assumptions 1–2, system (3) is equivalent to the following system

$$\begin{aligned} 0 &\leq p_k^i \perp \sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j + v_i + \varphi_k^i \geq 0, \quad \forall k = 1, 2, \dots, n \\ P_{\max}^i - \sum_{k=1}^n p_k^i &= 0, \quad v_i \text{ is free} \\ 0 &\leq \varphi_k^i \perp CAP_k^i - p_k^i \geq 0, \quad \forall k = 1, 2, \dots, n \end{aligned} \quad (4)$$

where $v_i = -\frac{1}{u_i}$, $\varphi_k^i = \frac{\gamma_k^i (\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j)}{u_i}$, and p^* is a solution of problem (1) if and only if there exist $v^* = (v^{1,*}, v^{2,*}, \dots, v^{m,*})^T \in R^m$ and $\varphi^* = ((\varphi^{1,*})^T, (\varphi^{2,*})^T, \dots, (\varphi^{m,*})^T)^T \in R^{mn}$ such that (p^*, v^*, φ^*) satisfies system (4).

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات