Computers & Operations Research 39 (2012) 1521–1529

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

A dynamic programming approach to solving the assortment planning problem with multiple quality levels

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ARTICLE INFO

Available online 5 September 2011

Keywords:
Assortment planning
Dynamic programming
Line search
Locational choice

ABSTRACT

While the assortment planning problem, in which a firm selects a set of products to offer, has been widely studied, several problem instances exist which have not yet been solved to optimality. In particular, we consider an assortment planning problem under a locational choice model for consumer choice with both vertical and horizontal differentiation. We present a combined dynamic programming/line search approach which finds an optimal solution when customer preference for the horizontal attributes are distributed according to a unimodal distribution. The dynamic program makes use of new analytical results, which show that high quality products will be distributed near the mode. This enables significant state reduction and therefore efficient solution times. Efficient computation times allow us to study the solution for a wide range of system parameters and thereby draw several managerial conclusions.

1. Introduction

Optimal solutions for the assortment planning problem with both vertical and horizontal differentiation under a locational choice model for consumer choice have only been partially characterized [6]. Here we use horizontal differentiation to refer to variety attributes, features for which different customers have different ideal choices but that do not affect the quality or value of the product, such as size or color. On the other hand, vertical differentiation refers to quality attributes; features that all customers prefer, all else being equal, such as higher resolution on a digital camera. Gaur and Honhon [2] find optimal solutions for the problem with no vertical differentiation (a single quality level). McElreath et al. [8] provide solutions for the problem with vertical differentiation, under the assumption that customer preference for the horizontal attribute is uniformly distributed. The authors also provide metaheuristics solutions [9] for the problem when consumer preference for the horizontal attribute follows a unimodal distribution. However, currently there is no method for generating optimal solutions to this problem. In this paper, we propose an optimal solution methodology for this problem by combining a dynamic program with a line search. The goal of this research is to provide computational solutions for this problem to allow for further exploration of the solution space. We will begin by briefly reviewing the assortment planning problem and the locational model for customer choice.

The assortment planning problem is the problem in which a retailer or manufacturer chooses a set of distinct products to offer to its customers with the goal of maximizing expected profit. This decision can have a substantial impact on the sales and profitability of the firm. Customers who find that their preferred products are not offered may depart without purchase. Beyond purely matching products to customers, firms may also wish to offer items with higher profit margins to customers who are willing to purchase them. Because the firm will pay some cost for each product it decides to offer, the total number of items which can be offered is constrained. The firm must balance customer demand with its own cost constraints—choosing a product line that is well matched with the characteristics of a firm's customer base will ensure the best possible revenue stream for that firm.

Assortment planning models largely differ in the way that customer purchasing decisions are modeled. The majority of work uses a discrete choice model for customer preference, often expressed as a Multinomial Logit (MNL) model. The MNL is a utility based model in which customers gain utility from purchasing 1 of N possible discrete alternatives. This model has advantages in that it is well established and relatively simple to implement. However, there are two primary shortcomings that motivate a separate approach. First, the MNL model implies that the potential pool of products should be discrete. In many cases (e.g. length and time) it would be impractical to discretize a large continuous space. Second, the MNL model requires that all available products be equal substitutes. That is, if a customer's preferred product is not available, he must be able

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doi:10.1016/j.cor.2011.08.023

to substitute for any other available product. This requirement is often illustrated through the use of the "red bus/blue bus" example (see Kök et al. [3]).

An alternative customer choice model is the locational choice model, first proposed by Lancaster [5]. In the locational choice model, customer preference is a continuous random variable distributed over some horizonal axis. An assortment consists of products which each have some location on this axis representing the preference attribute of that product. Customers receive utility from purchasing an item based on the price of the item and the distance between the product's attribute and that customer's preference. Utility declines linearly over this distance creating a maximum distance that a customer can be from a product and still receive positive utility. Because of this, substitution is controlled between products—a customer will only substitute a product which is located in the vicinity of his preference.

While this model is more appropriate in many cases, it introduces new computational challenges. In the assortment planning problem under discrete choice, a set of discrete products and associated probabilities are known and given. In the locational choice model, we must choose products from an infinite and continuous set. Further, the probability of an item being chosen is no longer independent, it depends on the locations of the other products in the assortment (two products located too close to one another can cannibalize each other's sales). This is further exemplified when we introduce vertical differentiation between products. With vertical differentiation, we allow the retailer to choose between multiple quality levels of products where higher quality products can provide higher profits for the retailer. Under discrete choice, this difference is insignificant—quality differences are implicity accounted for when defining the product set. In the locational choice model we must choose a quality level when locating a product, as each product is now defined by a location on the axis as well as a quality level. This results in a combinatorial problem which is inherently linked to our continuous location problem. For a more detailed review of the assortment planning problem see Kök et al. [3] or McElreath [7].

In this paper, we consider the assortment planning problem under locational choice allowing for horizontal and vertical differentiation using a unimodal distribution of customer preference. Presently, the optimal solutions for this problem are unknown. McElreath et al. [9] provide metaheuristic solutions to this problem but lack a tight upper bound to evaluate the quality of the solutions produced. We propose a combined dynamic programming/line search approach which leverages the relationship between the assortment planning problem and the knapsack problem. Using this approach, we provide optimal solutions to the assortment planning problem with locational choice and use the results to explore properties of the solutions. We will begin by formalizing the model, then describe the solution methodology and discuss the effects of parameters on the resulting assortment.

2. Model description

A firm operating in a make-to-order environment carries products that have both horizontal and vertical attributes. The firm wishes to select an assortment, a subset of products to offer, which maximizes profit. Customer preference for the horizontal attribute is distributed according to a unimodal distribution $F(z)$, $z \in \mathcal{B} \subset \mathcal{R}$. For the vertical attribute there are two quality levels of products available: low (L) and high (H). The firm will choose n (the number of distinct products she will offer) and, for each product j, (b_j,y_j) where b_j is the location of the product on the horizontal attribute space and y_j is the quality level, $j=1,\ldots,n$. Formally

$$b_j \in \Re \quad \text{and} \quad y_j \in \{L,H\}.$$

Let p_j be the selling price of a product which depends on the quality level, $p_j \in \{p_L,p_H\}$. The firm pays a unit cost, c, for each item sold and a fixed cost, K for each distinct product in the assortment. The intensity of demand is given by λ.

A customer who arrives at location z (representing his preference) and purchases a product at location b_j with quality y_j will receive utility given by

$$U(z,b_j,y_j) = u(y_j) - p_j - t\,|b_j - z|,$$

where

$$u(L) = v \quad \text{and} \quad u(H) = v + q.$$

We interpret v as the value obtained from purchasing a product and q represents the additional value from a high quality product; t is a travel cost for purchasing a product away from a customer's preference.

Each product has a coverage distance, $l_j \in \{l_L,l_H\}$, representing the maximum distance a customer's preference can be from a product's location and the customer can still receive positive utility. The coverage distance is given by

$$l_j = \frac{u(y_j) - p_j}{t}.$$

Each customer will choose the product which offers him the highest utility among all products offered in the assortment. The first choice interval, $[b_j^-, b_j^+]$ is an interval containing the locations of all customers who will choose product j as their first choice. In the optimal solution these intervals do not overlap, as shown by Mayorga [6]. Therefore, the first choice interval is given as

$$[b_j^-, b_j^+] = [b_j - l_j, b_j + l_j]. \tag{1}$$

We can then find the probability a random customer chooses product j, d_j as

$$d_j = F(b_j^+) - F(b_j^-). \tag{2}$$

The profit for a single product, Π_j is given by

$$\Pi_j(d_j) = (p_j - c)\lambda d_j - K. \tag{3}$$

The profit for the assortment is then given by the sum of the profits for all products:

$$\Pi = \sum_{j=1}^{n} \Pi_j.$$

In addition to the non-overlapping property, Mayorga [6] has shown that in the optimal solution coverage will be continuous. That is, $b_k^+ = b_{k+1}^- \forall k \in [1,n-1]$. As such, finding an optimal solutions requires two components, the starting position of the first product and the sequence of quality levels in the assortment. From this the remaining product locations are given by

$$b_j = b_1^- l_{y_j} + \sum_{i=1}^{j-1} 2l_{y_i}, \quad j = 1,\ldots,n. \tag{4}$$

While Mayorga [6] is able to characterize the optimal assortment under some conditions, in general the optimal solution to the problem remains unknown.

Therefore the optimization problem can be written as

$$\max_{(b_1,\mathbf{y})} \sum_{j=1}^{n} \Pi_j(d_j)$$
$$\text{s.t. } (1),(2),(3),(4).$$

Our solution approach consists of two combined components: a dynamic program to find an optimal sequence of products given the location of the first product ($\mathbf{y}^*(b_1)$), and a line search to find the optimal location of the first product in the assortment, b_1^*. We will first describe each component in general terms and then