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A two-phase hybrid heuristic algorithm for the capacitated location-routing problem

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ABSTRACT

In this paper, we propose a two-phase hybrid heuristic algorithm to solve the capacitated location-routing problem (CLRP). The CLRP combines depot location and routing decisions. We are given on input a set of identical vehicles (each having a capacity and a fixed cost), a set of depots with restricted capacities and opening costs, and a set of customers with deterministic demands. The problem consists of determining the depots to be opened, the customers and the vehicles to be assigned to each open depot, and the routes to be performed to fulfill the demand of the customers. The objective is to minimize the sum of the costs of the open depots, of the fixed cost associated with the used vehicles, and of the variable traveling costs related to the performed routes. In the proposed hybrid heuristic algorithm, after a *Construction phase* (first phase), a modified granular tabu search, with different diversification strategies, is applied during the *Improvement phase* (second phase). In addition, a random perturbation procedure is considered to avoid that the algorithm remains in a local optimum for a given number of iterations. Computational experiments on benchmark instances from the literature show that the proposed algorithm is able to produce, within short computing time, several solutions obtained by the previously published methods and new best known solutions.

1. Introduction

The *location routing problem* (LRP) includes two types of fundamental problems of the supply chain management: the facility location problem and the vehicle routing problem. The different aspects of these problems such as location, assignment and routing have been generally studied independently. This can be explained by considering that the location is a strategic decision which is taken for a long time frame, while the routing is an operational aspect which can be modified dynamically many times in a short time. However, it is well know that these decisions are interrelated. Indeed, the decision of locating a depot is often influenced by the transportation costs and vice versa (for more details see [1]). As a consequence, the LRP has become an interesting field of research.

This paper considers the *capacitated location-routing problem* (CLRP), i.e. the LRP with capacity constrains for both the depots and the routes. The CLRP can be defined as the following graph theory problem. Let $G=(V,E)$ be a complete undirected graph, in which $V=\{1,\dots,m+n\}$ is the vertex set and E is the edge set. Vertices $i=1,\dots,m$ correspond to the potential depots, each with a capacity W_i and an opening cost O_i. Vertices $j=m+1,\dots,m+n$ correspond to the customers, each with a nonnegative demand d_j. A set of homogeneous vehicles, each with capacity Q, is available at each depot. Each vehicle, when is used by a depot to perform a single route, causes a nonnegative fixed cost F. A nonnegative cost c_{ij} is associated with each edge $(i,j)\in E$.

The goal of the CLRP is to determine the depots to be opened, the customers and the vehicles to be assigned to each open depot, and the routes to be constructed to fulfill the demand of the customers with the minimum global cost, given by the sum of the set-up costs of the open depots, the costs of the used vehicles, and the costs of the edges traveled by the routes. The following constraints are imposed:

- each route must start and finish at the same depot;
- each customer is visited exactly once by a single route;
- the sum of the demands of the customers visited by each route must not exceed the vehicle capacity;
- the sum of the demands of the customers assigned to each depot must not exceed its corresponding capacity;
- connections between depots are not allowed.

The CLRP is NP-hard since it generalizes three well known NP-hard problems: the *capacitated facility location problem* (CFLP), the

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capacitated vehicle routing problem (CVRP), and the *multi depot vehicle routing problem* (MDVRP). The CFLP can be described as a CLRP with unlimited vehicle capacity (i.e. $Q=\infty$), vehicle fixed cost equal to zero (i.e. $F=0$), and infinite cost for the edges connecting any pair of customers (i.e. $c_{ij}=\infty$ for $i=m+1,\ldots,m+n$ and $j=m+1,\ldots,m+n$). The CVRP can be described as a CLRP with only one depot (i.e. $m=1$), and the MDVRP can be formulated as a CLRP with depot opening cost equal to zero (i.e. $O_i=0$ for $i=1,\ldots,m$).

In this paper, we propose a two-phase hybrid heuristic algorithm to solve the CLRP. The paper is organized as follows. The existing literature is described in Section 2. Section 3 details the proposed two-phase heuristic algorithm. Experimental results on the benchmark instances from the literature are presented in Section 4. Finally, conclusions and future research are given in Section 5.

2. Literature review

Few surveys on location-routing problems have been presented in the literature. Min et al. [2] proposed a classification for the LRP based on the solution methods, and the problem perspectives. The most recent classification, proposed by Nagy and Salhi [3], is based on eight different aspects. This hierarchical taxonomy provides a more integrated view of the LRP literature.

Different mathematical formulations with two and three indices have been proposed for the LRP and the CLRP. Three-index formulations for the LRP were introduced by Perl and Daskin [4] and Hansen et al. [5], and for the CLRP by Prins et al. [6]. Two-index formulations for the CLRP have been proposed by Laporte et al. [7], Contardo et al. [8] and Belenguer et al. [9]. These exact approaches can consistently solve to proven optimality instances with less than 100 customers. For this reason, heuristic algorithms have been proposed to solve large CLRP instances.

Nagy and Salhi [3] classified these algorithms as sequential, iterative, hierarchical, and clustering based methods. Sequential methods usually solve the facility location problem, and then the corresponding routing problem for each open depot (see, e.g. [10]). According to Salhi and Rand [11], this type of approach avoids an important feedback between the two subproblems.

On the other hand, iterative methods solve both subproblems in an iterative way providing a feedback between the two subproblems. In these methods, the CLRP is tackled either by solving the corresponding routing problem without considering the location decisions and assigning one depot for each cluster of customers, or by solving the facility location problem and performing at least one route for each open depot. Tuzun and Burke [12] proposed a two-phase tabu search approach that iterates between the location and the routing phases in order to search better solutions for large instances. In this work, results for instances with up to 200 customers have been reported.

Prins et al. [6] proposed a two-phase algorithm which exchanges information between the location and routing phases. In the first phase, the routes and their customers are aggregated into super customers, and the corresponding capacitated facility location problem is solved by using a Lagrangean relaxation technique. In the second phase, a *granular tabu search* (GTS) procedure (see [13]) with one neighborhood was used to solve the resulting multi depot vehicle routing problem. At the end of each iteration, information about the promising edges is recorded to be used in the following phase.

Hierarchical methods solve the CLRP by using a hierarchical structure. First, the facility location problem is solved as the main problem, and then, the subsequent routing problem is solved as the subordinate problem. The location problem is solved in an approximate way by applying at each step a subroutine that solves the corresponding routing problem. Interested readers are referred to Albareda-Sambola et al. [14] and Melechovský et al. [15].

Cluster based methods for the CLRP were proposed by [16]. In this work, in the first phase the customer set is split into clusters according to the vehicle capacity. In the second phase, a *Traveling Salesman Problem* (TSP) is solved for each cluster. Finally, in the final phase, the TSP circuits are grouped into super nodes for solving the corresponding capacitated facility location problem.

Other heuristics for the CLRP have been proposed by Prins et al. [17]. In this work, a greedy randomized adaptive search procedure (GRASP), with a learning process and a path relinking strategy, has been proposed. A randomized version of the Clarke and Wright algorithm (proposed by [18] for the CVRP) is applied during the GRASP phase. In addition, a learning process is implemented to choose the correct depots. A path relinking strategy is then used as post optimization procedure to generate new solutions. The same authors ([19]) proposed a memetic algorithm with population management.

Recent heuristics for solving the CLRP were developed by Yu et al. [20] and Duhamel et al. [21]. In the first work, a simulated annealing procedure (SA) based on three random neighborhood structures has been proposed. In the second work, Duhamel et al. [21] proposed a successful method based on a hybridized GRASP with a evolutionary local search (ELS) procedure.

3. Description of the proposed algorithm

This section presents a two-phase hybrid heuristic algorithm (2-Phase HGTS) developed for solving the CLRP. The main body of the proposed algorithm consists of two major phases: *Construction phase* and *Improvement phase*. In the *Construction phase*, the goal is to build an initial feasible solution using an *Initial hybrid procedure* followed by a *Splitting procedure* to minimize the routing cost. In the *Improvement phase*, a modified GTS procedure, which considers several diversification steps, is applied to improve the quality of the current solution. Whenever no improvement is obtained within $N_{pert}\times n$ iterations (where N_{pert} is a given parameter), the algorithm tries to escape from the current local optimum by applying a randomized *perturbation procedure*. In addition, a *procedure VRPH*, based on the library of local search heuristics for the VRP proposed by [22], is introduced as a general improvement routine.

The key-point for the success of the proposed algorithm is the location of the correct depots in the *Construction phase*. Since the most critical decisions of the *Improvement phase* are those concerning the opening and closing of the depots, a proper location of the depots is able to reduce the search space for the *Improvement phase* from a CLRP to a MDVRP. The previously mentioned procedures are described in more detail in the following subsections.

3.1. Procedure VRPH

Groer et al. [22] have recently proposed a software library containing fast local search heuristics for finding good feasible solutions for the CVRP. The standard library offers four different routines:

- *vrp_initial*: This routine uses a variant of the Clarke-Wright algorithm, proposed by Yellow [23], to generate initial solutions for the CVRP;
- *vrp_rtr*: This routine is an implementation of the record-to-record travel metaheuristic proposed by Li et al. [24];