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Discrete dynamic programming with outcomes in random variable structures

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Abstract

Multiobjective approach is the common way of generalization single-criterion dynamic programming models. Another way is to consider partially ordered criteria structures. That approach is rather rare. The aim of the paper is to present such a model. Generalization of Bellman's principle of optimality is employed to create a forward procedure to find the set of all maximal elements. As this set is usual large, the second problem under consideration is to find its subsets. To reduce the number of solutions presented to decision maker we propose to apply a family of narrowing relations. That approach is similar to scalarization in multiobjective programming. Ordered structures of random variables based on mean–variance, stochastic dominance and inverse stochastic dominance are considered. Numerical illustration is given at the end of the paper.

Keywords: Dynamic programming; Discrete optimization; Multiobjective decision making; Stochastic orders

1. Introduction

The real life decision problems are often characterized by conflicting objectives and multiple stages. One way of modelling such situations is multiobjective approach. Many research papers have been published in this field in recent years. Surveys of research on multiobjective dynamic programming were presented by Li and Haimes (1989) and Trzaskalik (1997, 1998). Many theoretical and application directions of research have been presented. For instance, Klotzler (1978) derived the method based on dynamic programming recurrence computations of optimal values for discrete dynamic systems and vector-valued objective function. Yu and Seiford (1981) considered vector optimization problems in the context of dynamic multistage

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systems. Impact analysis was presented by Gomide and Haimes (1984). Multiobjective shortest path problems were considered by Henig (1985a), Warburton (1987), and Getachew (1992). Time dependency in multiobjective dynamic programming was presented by Kostreva and Wiecek (1993) and Kostreva and Lancaster (2002).

Various scalarization and generating techniques can be used to convert multiobjective dynamic programming problem into scalar-objective optimization problems. Weighted-sum approach and ε-constraint method can be found in Tauxe et al. (1979). An interactive trade-off approach was applied in Hu and Wu (1989). An algorithm for decomposition of the parametric space by using the weighted norm was presented in Abo-Sinna and Hussein (1994). Multiobjective trajectory optimization based on reference trajectories was introduced in Wierzbicki (1979). Dynamic goal programming models and lexicographic goal programming models with application of period and multi-period target values were presented in Trzaskalik (1997) and Caballero et al. (1998). Lexicographic multiobjective dynamic programming was described by Szidarovszky and Duckstein (1986). Opricovic (1993) developed compromise multiobjective dynamic programming method.

Multiobjective dynamic programming can be considered as generalization of the single-criterion dynamic programming, introduced by Bellman (1957). Another way of generalization is taking into consideration returns in partially ordered criteria set. Mitten (1974) described a method of solving a variety of multistage decision problems in which the real value objective function was replaced by preference relation. Sufficient conditions were given to ensure that the recursive dynamic programming procedure yields the optimal sequence of decisions. Sobel (1975) extended Mitten's results to infinite horizon for deterministic and stochastic problems. Preference order dynamic programming was described by Steinberg and Parks (1979). Henig (1985b) defined a general sequential model with returns in partially ordered set. Discrete dynamic programming with partially ordered criteria set was also considered by Trzaskalik and Sitarz (2002). Principle of optimality and backward numerical procedure were derived.

The present paper is the continuation of our previous one, mentioned above. The aim of the paper is to describe finite, discrete model and forward procedure of finding the set of all maximal elements, based on Bellman's principle of optimality as well as to introduce narrowing relations and their applications for partially ordered structures of random variables. The organization of the paper is as follows. In Section 2 discrete dynamic decision process is defined. In Section 3 Bellman's principle of optimality is applied to develop forward procedure to solve the problem of finding all maximal values of the process. The problem considered in Section 3 is to find its subsets. In Section 4 ordered structures of random variables are considered and proposed. Narrowing methods in stochastic dominance, inverse stochastic dominance and mean–variance models are developed. Numerical illustration is given in Section 5. Discussion and concluding remarks are included in Section 6.

2. Dynamic programming procedures in partially ordered structure

We consider a finite discrete dynamic process. Markovian property is assumed. The exemplary process structure is presented in Fig. 1.

For a given process the graph of the process is assigned as follows: the nodes correspond to the states of the process, the arcs correspond to the decisions.

The model presented below may be used for solving decision problems with stochastic parameters. One of the applications is a problem of inventory control which can be described as follows. We consider finite process divided into periods. The amount of inventory at the beginning of the process is known. Maximal size inventory supply at each period and maximal size of storage at each period are also known. Cost of purchase for each period is formed at random. We are to establish such an inventory supply plan that the cost of purchase and the cost of storage will be minimized. The demand is to be fully satisfied. As