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Ant algorithms and simulated annealing for multicriteria dynamic programming

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Abstract

The paper presents a comparison of ant algorithms and simulated annealing as well as their applications in multicriteria discrete dynamic programming. The considered dynamic process consists of finite states and decision variables. In order to describe the effectiveness of multicriteria algorithms, four measures of the quality of the nondominated set approximations are used.

Keywords: Multicriteria programming; Dynamic programming; Ant colony optimization; Simulated annealing; Hybrid algorithms

1. Introduction

The theory of dynamic programming, first used by Bellman in the early 1950s, resulted in the Principle of Optimality which helps to solve problems which have a separated and monotonic criterion function [1]. Afterwards some generalizations of Bellman's Principle of Optimality appeared in different structures: in lattices [2], in partially ordered sets [3], and in ordered structures [4]. However, the method of solving problems, based on the Bellman's Principle of Optimality, is neither effective for multidimensional problems nor for multicriteria problems (so-called "curse of dimension" which occurs even in single criterion dynamic programming). Bearing this in mind, we should look for different methods of solving dynamic programming problems. One of the possibilities is to use metaheuristic algorithms. Among the wide class of such algorithms are ant algorithms [5] and simulated annealing (SA) [6], which will be examined in this paper in the view of their efficiency in dynamic programming. The idea of ant algorithms comes from solving the Travelling Salesman Problem (TSP) which is similar to the discrete dynamic programming problem (these two problems are presented in the form of a graph). Therefore, it is natural to try to transfer positive results of ant algorithms to dynamic programming (single or multicriteria).

In the second section, the considered multicriteria dynamic programing problem is described. This was presented in detail in Trzaskalik and Sitarz [4,7]. Moreover, we introduce the test problem in order to verify the developed algorithms. The third section describes ant algorithms and their modifications. It presents ideas on how to apply ant algorithms to the discrete dynamic programming problem. The next section describes SA and its modification in multicriteria optimization. The numerical results of the proposed algorithms are presented in the last section.

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2. Discrete dynamic problems (DDP)

2.1. Model of the problem

We consider the multiperiod dynamic process consisting of T periods in which there are the following sets defined for $t = 1, \ldots, T$:

Y_t—set of all feasible state variables at the beginning of the period t.
Y_{T+1}—set of all states at the end of the process.
$X_t(y_t)$—set of all feasible decision variables for the period t and the state $y_t \in Y_t$.

We assume that the above sets are finite. Then, we denote the state transformation with

$$\Omega_t : D_t \to Y_{t+1},$$

where $D_t = \{r_t = (y_t, x_t) : y_t \in Y_t \wedge x_t \in X_t(y_t)\}$ is a set of period realizations (y_t, x_t) for the period t.

Moreover, we denote a set of all feasible process realizations with D:

$$D = \{r = (r_1, \ldots, r_T) : \forall_{t\in\{1,\ldots,T\}} y_{t+1} = \Omega_t(y_t, x_t) \wedge x_t \in X_t(y_t)\}.$$

For $t = 1, \ldots, T$ there are defined period criteria functions $f_t : D_t \to \boldsymbol{R}^N$ where $\boldsymbol{R}^N$ is the set of N-dimension real vectors. Using the function f_t, we receive the multiperiod criteria function $F : D \to \boldsymbol{R}^N$ which is the sum of period functions:

$$F = f_1 + f_2 + \cdots + f_T.$$

We call the realization $r^* \in D$ efficient when $F(r^*)$ is a nondominated vector (in the sense of minimization) in the set $F(D)$, that is

$$\sim \exists_{r\in D} F(r) \leqslant F(r^*) \wedge F(r) \neq F(r^*).$$

Remark 1. The general definition of dynamic programming, with criteria function values defined in a partially ordered set, is included in the work of Trzaskalik and Sitarz [7]. There is also a numeric algorithm which gives efficient solutions based on Bellman's Principle of Optimality.

Remark 2. We may attribute an adequate graph for every discrete multiperiod decision process in the following way: the nodes of the graph correspond with the process states, and the edges of the graph with decisions. A path linking the initial node of the graph with the final node corresponds with every process realization. There is a deeper analysis of the relations between graphs and discrete dynamic processes in the work of Trzaskalik [8].

Remark 3. In a special case, when the dimension of the value criteria function space is equal to one, we obtain a single criterion problem.

2.2. *Test problem*—MDDP(T, 2)

The name of the test problem follows from Multicriteria Discrete Dynamic Problem with T periods and two states. We will consider the dynamic process composed of T periods and two states in every period:

$$Y_t = \{0, 1\} \quad \text{for } t = 1, \ldots, T+1,$$

$$X_t(y_t) = \{0, 1\} \quad \text{for } t = 1, \ldots, T \text{ and } y_t \in Y_t,$$

$$\Omega_t(y_t, x_t) = x_t \quad \text{for } y_t \in Y_t \text{ and } x_t \in X_t(y_t).$$

The period criteria functions $f_t : D_t \to \boldsymbol{R}^2$ in MDDP $(T, 2)$ are the same in every period $t = 1, \ldots, T$, and they have the following form:

$$f_t(1, 1) = (4, 4), \quad f_t(1, 0) = (1, 3), \quad f_t(0, 1) = (6, 2), \quad f_t(0, 0) = (5, 1).$$

The dynamic process is illustrated in Fig. 1.