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The valuation of catastrophe bonds with exposure to currency exchange risk☆

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ABSTRACT

In this paper, we present a new model that takes an arbitrage approach to the valuation of catastrophic risk bonds (CAT bonds). The model considers the sponsor's exposure to currency exchange risk and the risk of catastrophic events. We use a jump-diffusion process for catastrophic events, a three-dimensional stochastic process for the exchange rate and domestic and foreign interest rates, and a hedging cost for the currency risk to derive a semi-closed-form formula for the CAT bond price. We also extend to three factors Joshi and Leung's (2007) Monte Carlo simulation approach to obtain numerical results showing the following: in addition to catastrophic risk, the CAT bond price is affected mainly by the volatility of the exchange rate and its correlations with domestic and foreign interest rates. The first two factors have a negative impact while the third has a positive impact.

1. Introduction

With average annual temperatures rising and the frequency and severity of natural disasters (floods, earthquakes, hurricanes, tornados, etc.) increasing in all parts of the world, the dire environmental impacts and vital socio-political and economic problems associated with global warming are alarming. Catastrophic risks associated with climate change are of great importance to financial markets, institutions and other quarters, including government, whose stakes are greatly affected by natural disasters.

Following Hurricane Andrew in 1992, heavily hit insurance and reinsurance companies inflicted with losses in billions of U.S. dollars turned to capital markets to search for alternative sources of funding to indemnify natural catastrophe and disaster victims. Since then, there has been a steady increase in new financial products traded on over-the-counter markets. Through securitization, insurance-linked (or risk-linked) securities transfer the catastrophic risk to capital market investors. Indeed, according to Cummins (2008), "risk-linked securities are innovative financing devices that enable insurance risk to be sold in capital markets, raising funds that insurers and reinsurers can use to pay claims arising from mega-catastrophes and other loss events."

The catastrophic risk bond (or catastrophe bond, or CAT bond) is the most prominent type of risk-linked security, in terms of both importance and volume. It is issued by insurance and reinsurance companies to share and spread natural disaster loss risks with other investors. CAT bonds are bonds where the coupon and principal payments are stopped after the occurrence of a triggering event. If an index of a natural risk or an insurance industry loss reaches a predetermined threshold, the holder of the CAT bond loses either his coupons or the face value, or both, depending on the agreed upon contract. CAT bond are issued through a *Special-Purpose Vehicle* (or SPV) located in a fiscally favourable zone such as the Cayman Islands. Given their low correlation with wider financial risk, investments in CAT bonds have proven resilient to the effects of recent financial crises and have grown substantially. This setup leads to a financial instrument linked solely to catastrophes, because the sponsor's credit risk is eliminated. However, CAT bonds are subject to other risks, including the risk of counterparty arising from a rate swap, risk of interest rates inherent to all bonds, and the currency exchange risk which is the subject of this paper.

The majority of CAT bonds are denominated primarily in U.S. dollars (others are Euro or Yen-denominated), thereby exposing the issuer

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whose national currency is not the U.S. dollar to an exchange risk that he is obliged to assume. Naturally, this risk should be compensated and can be modelled by introducing a currency hedging cost in the bond pricing. The issuer takes a long position on a call on his currency to lock in the exchange rate at the maturity of the CAT bond and exercise it if both the CAT bond is triggered and the exchange rate is not favourable at the trigger time. To the best of our knowledge, the only contribution on this topic has been made by Poncet and Vaugirard (2001), who take into account the currency exchange risk by considering options sold by the sponsor on his currency to hedge against this risk. Assuming a non-catastrophic environment where the loss index is a diffusion process, these authors derive an explicit formula for the bond price. Their results indicate that the currency exchange risk has a negative effect – albeit weak – on the CAT bond price, compared to the natural risk.

To quantify the exchange rate impact on a CAT bond issuer, this paper develops a new valuation model for CAT bonds that considers both catastrophic events and exchange rate risks ignored in the extant literature. Like Vaugirard (2003a), we model the catastrophic risk in a jump diffusion process that characterizes the catastrophe index dynamics. Furthermore, by introducing a model with three factors to describe the exchange rate, as in Hilliard, Madura, and Tucker (1991), we depict its dynamics in a richer framework. Our contribution consists of the derivation of a quasi closed-form formula for the CAT bond price and an extension to three factors of Joshi and Leung's (2007) Monte Carlo simulation approach that combines importance sampling and Brownian bridge techniques between catastrophic events.

The results of our numerical experiments show that the currency exchange risk is small compared to the risk of natural catastrophes. Both the volatility of the exchange rate and the correlation between the exchange rate and the domestic interest rate have a negative impact on the CAT bond price. Conversely, the correlation between the exchange rate and the foreign interest rate exhibits a positive relation with the CAT bond price.

The paper is organized as follows. After a literature review in Section 2, we derive our valuation model and formulas in Section 3 and describe the Monte Carlo simulation approach in Section 4. We present numerical comparative statics results in Section 5 and conclude in Section 6. In Appendix A, we discuss some background and structured finance characteristics of CAT bonds and in Appendix B, we illustrate our simulation procedure with a numerical example.

2. Literature review

2.1. CAT bond valuation

2.1.1. Incompleteness of markets in the presence of natural catastrophes

Natural catastrophes are often modelled by superposing a jump process on the underlying process, whether it represents a parametric index or losses. In this case, financial markets are incomplete. Therefore, it is not possible to devise a portfolio replication strategy to price the derivative asset, and the equivalent martingale measure is not unique. Two approaches have been developed to address the situation. The first approach by Merton (1976) assumes that the risk associated with the jumps can be eliminated entirely by diversification; it is idiosyncratic or unsystematic. We can then say that investors are "risk-neutralized" with respect to the risk of natural catastrophes. With this assumption, the equivalent martingale measure is unique and arbitrage pricing is warranted. The second approach involves an equilibrium model. Typically, in this approach, we must assume a utility function for the investor and perform the valuation by maximizing expected utility under the real probability measure.

2.1.2. Risk-neutralized CAT bond valuation

The Merton (1976) approach leads to the existence and uniqueness of an equivalent martingale measure that entails the absence of arbitrage opportunities. The first studies of this kind on CAT bonds ignore the catastrophic risk, as in the case of Poncet and Vaugirard (2002). They use a simple diffusion process for the catastrophe index under a stochastic interest rates regime. Although somewhat unrealistic, this model nonetheless offers a closed-form expression for the CAT bond price. Vaugirard (2003a) extends the model by considering natural catastrophes in a jump diffusion process. Vaugirard (2003b) then replaces the jump diffusion process by a mean reverting process for his CAT bond pricing model. In a different vein, Lee and Yu (2002) propose an insurance model for valuing CAT bonds using a compound Poisson process for the loss dynamics. They study the impact of default risk, moral hazard and basis risk on the CAT bond price. For the sake of realism, some studies assume the frequency of catastrophes to be stochastic, as in the case of Baryshnikov, Mayo, and Taylor (2001). Dassios and Jang (2003) use a "doubly stochastic" Poisson process and Albrecher, Hartinger, and Tichy (2004) provide an algorithm using a quasi-Monte Carlo simulation to solve this model. Hainaut (2010) follows this approach, but introduces a seasonal component for the intensity.

Jarrow (2010) presents an alternative methodology. Exploiting the analogy between catastrophe risk and default risk, he adapts a simple model for pricing credit derivatives to value CAT bonds based on two input parameters: the probability of a catastrophe occurring per unit of time and the percentage of loss imparted to the catastrophe. This model is remarkably simple but requires empirical estimates of these two parameters that may be difficult to obtain owing to the lack of liquidity of traded CAT bonds.

2.1.3. Equilibrium models

To deal with the problem of market incompleteness in the presence of catastrophic risk, Cox and Pedersen (1995) propose a model based on the "representative agent" paradigm that postulates a utility function for the investor and a consumption process. The price of the CAT bond is obtained by maximizing the investor's expected utility. Egami and Young (2007) revisit this methodology to price structured CAT bonds composed of two tranches, *junior* and *senior*. Reshetar (2008) evaluates multi-catastrophe CAT bonds that specifically consider the risk of terrorist attacks.

2.2. Modelling the exchange rate risk

Exchange rate risk modelling can be found in the literature on the valuation of currency options. Garman and Kohlhagen (1983) extend the Black–Scholes formula to currency options in which the foreign interest rate plays the same role as a dividend rate. Grabbe (1983) develops a model where the prices of domestic and foreign bonds are assumed to follow geometric Brownian motions and then obtains a closed-form formula. Amin and Jarrow (1991) tackle the same problem using the framework of Heath, Jarrow, and Morton (1992). Hilliard et al. (1991) provide a generalization of the previous models for stochastic domestic and foreign interest rates correlated with the exchange rate. They show that the model where interest rates are stochastic yields better performance than the model with constant rates for estimating the price of options on currencies.

Other authors note the importance of introducing stochastic volatility in the exchange rate. Hakala and Wystup (2002) adapt Heston's volatility model to exchange rate options. Haastrecht, Lord, Pelsser, and Schrager (2009) and Grzelak and Oosterle (2010) consider a four-factor model with stochastic interest rates and volatilities, but do not find closed-form formulas for valuing options on currency.

3. Valuation model

3.1. Analytical framework

3.1.1. Dynamics of the economy

We model the exchange rate S_t with a geometric Brownian motion and the domestic and foreign interest rates r_d and r_f each according to