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Expected returns, risk premia, and volatility surfaces implicit in option market prices

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ABSTRACT

This article presents a pure exchange economy that extends Rubinstein (1976) to show how the jump–diffusion option pricing model of Merton (1976) is altered when jumps are correlated with diffusive risks. A non-zero correlation between jumps and diffusive risks is necessary in order to resolve the positively sloped implied volatility term structure inherent in traditional jump diffusion models. Our evidence is consistent with a negative covariance, producing a non-monotonic term structure. For the proposed market structure, we present a closed form asset pricing model that depends on the factors of the traditional jump–diffusion models, and on both the covariance of the diffusive pricing kernel with price jumps and the covariance of the jumps of the pricing kernel with the diffusive price. We present statistical evidence that these covariances are positive. For our model the expected stock return, jump and diffusive risk premiums are non-linear functions of time.

1. Introduction

Das and Sundaram (1999) illustrate that neither jump–diffusion nor stochastic volatility option pricing models are capable of producing all features of the implied volatility surfaces calibrated from equity and currency option prices. In particular, traditional jump diffusion models produce only positively sloped implied volatility term structures which contrasts markedly with empirical non-monotonic and frequently negatively sloped implied volatility term structures, see Mixon (2007) and Camara (2009) among many others.

In our generalized jump diffusion option pricing model, the covariance between the diffusive price and price jumps, ρ_{sy}, plays an important role that is not shared by any existing jump–diffusion option pricing model. A jump diffusion model with non-zero ρ_{sy} can generate increasing, decreasing, and non-monotone term structures of implied volatilities. This is relevant since as highlighted by Das and Sundaram (p. 213), "the term structure of implied volatilities of at-the-money forward options in the jump–diffusion [model of Merton (1976)] is always an increasing function of the time-to-maturity. This puts the model at odds with the data, which suggests that decreasing or non-monotone term structure patterns frequently arise in practice". We illustrate that $\rho_{sy} \neq 0$ is a necessary condition for negatively sloped and/or non-monotonic implied volatility term structures, that $\rho_{sy} < 0$ is necessary to produce a convex implied volatility term structure with inflection point at finite time till expiration, and that the time till expiration at which the slope of the volatility term structure changes sign is proportional to the reciprocal of ρ_{sy}. Consistent with traditional jump diffusion models, we illustrate that $\rho_{sy} = 0$ is a necessary condition for a positive monotonic relationship between implied volatility and time till expiration.

Parameter estimates of our option pricing model were obtained by calibrating the model to a sample of S&P 500 index options. In our sample period, January 1996–April 2006, the ρ_{sy} is negative and significantly different from zero. The negative correlation produces a U shaped term structure of implied volatilities. Further, the negative correlation is consistent with higher price levels associated with downward price jumps.

The equity risk premium of our equilibrium asset pricing model can be decomposed into a diffusive risk premium and jump risk premium. The diffusive risk premium arises from both the covariance of the diffusive price with the diffusive pricing kernel, and the covariance of the diffusive price with the jumps of the pricing kernel. The new term, that takes into account the covariance of the diffusive price with the jumps of the pricing kernel, adds 4.9% to the annual diffusive risk premium. The jump risk premium arises from both the covariance of price jumps with the jumps of the

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pricing kernel, and the covariance of price jumps with the diffusive pricing kernel. Our evidence is consistent with an annual equity jump risk premium of 12.12%. Even in approximated terms, the expected stock return, the jump risk premium and the diffusive risk premium are nonlinear functions of time completely determined by non-zero covariances. In equilibrium, all four sources of the equity risk premium affect asset prices, and the two new factors in this article play a determinant role in the shape of the implied volatility *smiles* and *sneers*.

Empirical evidence that investors require a jump risk premium in addition to the diffusion risk premium is provided by Pan (2002), Eraker (2004), Santa-Clara and Yan (2010), Broadie et al. (2007), but these authors ignore the risk premium born with the correlation between Brownian motions and jumps. According to our empirical results, the effect of the covariance between the diffusive pricing kernel and price jumps represents around 45% of the jump risk premium.

The Merton (1976) model assumes that jumps are idiosyncratic risk and therefore uncompensated by a risk premium. This work was extended by Naik and Lee (1990), and Amin and Ng (1993) who derived option pricing formulae of the Merton's type in the presence of systematic jump risk resulting from simultaneous jumps in the stock price and the pricing kernel. Recently, theoretical advances to the model have been made by Duffie et al. (2000) who extend the theory by allowing jumps in volatility and Santa-Clara and Yan (2010) who allow the jump intensity to follow its own stochastic process.

A standard assumption of all previous jump–diffusion literature is that the Brownian motions and the jumps are independent. However, Santa-Clara and Yan (2010) present empirical evidence showing that jump risk is correlated with the stock index. This suggests that the stock price level is not independent of the size of the price jump. Duffie et al. (2000) also remark that one potential explanation for some stylized facts in option markets might be the fact that option pricing models unnecessarily restrict the correlations of the state variables. These works help to motivate our assumption that Brownian motions and jumps are correlated.

The consumption asset pricing model for our jump diffusion economy is derived assuming a representative agent with power utility function optimizing expected utility of consumption through time in an environment characterized by jump–diffusion processes with simultaneous random jumping times for aggregate consumption and stock price. Our evidence is consistent with a coefficient of proportional risk aversion of 6.55, which is within the range of estimated parameter values by Bliss and Panigirtzoglou (2004). The equilibrium interest rate is determined not only by the usual parameters of the traditional jump–diffusion models with systematic jump and diffusion risks, but also by the covariance between the diffusive pricing kernel and the jumps of the pricing kernel. This, in general, leads to a non-flat term structure of interest rates.

Although the Heston stochastic volatility model is considered to be a closed form model, its implementation requires inversion of the characteristic function of the log price of the underlying asset. Inverting the characteristic function requires numeric evaluation of complex integrals to approximate the probability of the option finishing in the money on the option's expiration date. The inversion may be accomplished with the transformation method as in Duffie et al. (2000). The formula for affine jump–diffusions is obtained by the Lévy inversion formula typically computed numerically with parameters determined by Riccati ordinary differential equation with boundary conditions. Our closed form option pricing model has the significant advantage that option values are determined explicitly by the equilibrium pricing relationship.

We derive a closed form consumption capital asset pricing model. The equity risk premium of this model has four distinct factors as previously remarked. Our general jump–diffusion model, which considers a full covariance structure of the underlying uncertainty to the pricing kernel and stock price, applies the technique of pricing by substitution in equilibrium introduced by Rubinstein (1976) to extend the Merton's (1976) option pricing model and derive formulae of the Merton's type. In section six of this paper we present the derivation of Merton's model under more general assumptions. Our general option pricing equation depends not only on the parameters of the traditional option pricing models with systematic diffusion risk and systematic jump risk, but also on the ρ_{sy}, the covariance between the diffusive pricing kernel and price jumps, the covariance between the diffusive price and jumps in the pricing kernel, and the covariance between the diffusive pricing kernel and jumps in the pricing kernel.

The remainder of the paper is organized as follows. Section 2 presents the standard Euler equations. Section 3 derives our general jump–diffusion model and illustrates the necessary and sufficient condition to generate a U-shaped implied volatility term structure. Section 4 presents our empirical results providing evidence consistent with a negative correlation between stock price diffusion and jumps.

We consider several special cases of our general jump diffusion model. For example, it is possible to construct a model of asset prices with systematic jump risk even if aggregate consumption and the pricing kernel do not jump. In Section 5 we study the roles played by the covariance between the diffusive pricing kernel and price jumps. Section 6 specializes the model by restricting the covariance structure of the underlying uncertainty to aggregate consumption and the stock price. In this section it is assumed that aggregate consumption follows a jump–diffusion process while the stock price follows a geometric Brownian motion. Section 7 extends the Merton (1976) option pricing model assuming that the stock price level and stock price jumps are correlated. Here, we illustrate the roles played by the covariance between the price level and price jumps. In Section 8, we present the conclusions of the paper.

2. The Euler equation

The results of this paper are obtained in economies that extend the pure exchange economy of Rubinstein (1976). There is a representative agent who maximizes his expected utility of consumption when he makes his consumption and investment decisions. To derive the asset valuation relationships we assume that this representative agent is nonsatiated and risk averse. These well known results are presented briefly and later used in the proofs of our results.

The representative investor maximizes:

$$E\left[\sum_{t=0}^{T} U_t(C_t)|\mathcal{F}_0\right], \tag{1}$$

where E is the expectation operator, $U_t(C_t)$ is the utility function of consumption at date t, and $\mathcal{F}_0$ is the information set available to the investor at date $t = 0$.

In this article, we price a riskless bond, a stock, and a call option, with terminal payoffs at time T of \$1, S_T, and $(S_T - K)^+$, where K is the exercise price of the option.[1] The current price of an arbitrary asset, P_0, is given by the Euler equation:

$$P_0 = E\left[\frac{U'(C_T)}{U'(C_0)}\phi(S_T)|\mathcal{F}_0\right], \tag{2}$$

[1] The price of the put can be obtained by using the put-call parity.