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Impact of uncertainty in expected return estimation on stock price volatility

Zvonko Kostanjcar [*], Branko Jeren, Zeljan Juretic

Department of Electronic Systems and Information Processing, Faculty of Electrical Engineering and Computing, University of Zagreb, 10000 Zagreb, Croatia

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ABSTRACT

We investigate the origin of volatility in financial markets by defining an analytical model for time evolution of stock share prices. The defined model is similar to the GARCH class of models, but can additionally exhibit bimodal behaviour in the supply–demand structure of the market. Moreover, it differs from existing Ising-type models. It turns out that the constructed model is a solution of a thermodynamic limit of a Gibbs probability measure when the number of traders and the number of stock shares approaches infinity. The energy functional of the Gibbs probability measure is derived from the Nash equilibrium of the underlying game.

1. Introduction

The motivation for this paper is to provide insight into the origin of volatility of stock share prices. The term volatility in financial markets is used as a measure of stock share price fluctuation variability [1]. To provide insight into the origin of price volatility it is necessary to understand why stock prices change. The traditional answer, within the theory of efficient markets [2,3], is that prices move because some new information becomes available, leading to a revision of the expectations of market participants. Under the assumption that this view is valid and in the absence of noise traders, there would be evidence that true investment value changes sufficiently through time to justify the price changes. Robert Shiller considered three indicators of change in true investment value of the aggregate stock market in the United States from 1871 to 1986: changes in dividends, in real interest rates, and in a direct measure of intertemporal marginal rates of substitutions [4]. According to his research, the contribution of these indicators is insufficient to completely explain volatility of stock market prices [4,5]. The essential meaning of Efficient Market Hypothesis (EMH) is that if the market price were predictable, then these opportunities would be exploited to make a gain so that such opportunities would disappear in a competitive and efficient market [6,7]. Furthermore, according to the weak-form version of EMH, at any given time, the price of an asset fully reflects all available historical information. The existence of an autocorrelation between distant observations breaks the market efficiency because past prices can help to predict future prices; i.e., correlated markets allow for arbitrage opportunities [8]. Di Matteo et al. have found that emergent markets have greater correlation than developed markets, suggesting more predictability [9,10]. Thus, emergent economies seem to be less efficient than developed ones. More recently, a negative correlation between stock market efficiency and predictability was experimentally assessed [11,12].

Today, many empirical studies point out the fact that it is difficult to justify the observed level of variability in stock prices by variations in fundamental economic factors [5,13]. In particular, the occurrence of large price jumps is not always explainable by the arrival of new information on the market [14,15]. Recently, Joulin et al. [16] studied price jumps. They concluded that most large jumps are not related to any broadcast news, even if one extends the notion of news to a (possibly

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[*] Corresponding author. Tel.: +385 1 6129 970; fax: +385 1 6129 652.
E-mail addresses: zvonko.kostanjcar@fer.hr (Z. Kostanjcar), branko.jeren@fer.hr (B. Jeren), zeljan.juretic@fer.hr (Z. Juretic).

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endogenous) collective market or sector jump. They found that the volatility pattern around jumps and around news is quite different, confirming that these are distinct market phenomena. Jumps are usually followed by increased volatility, whereas news tends on average to be followed by lower volatility levels. From data presented it seems that most of the volatility arises from trading itself, through the very impact of trades on prices. Several researchers during the years have reached the same conclusion, for example Cutler et al. in Ref. [17].

A few simplified market models were introduced in an attempt to reproduce and explain the impact of trade on prices. Maslov proposed a model based on the assumption that there are two kinds of market participants [18,19]. One type of investor places their orders at a prescribed price and a trade occurs as soon as there is anybody accepting that price. The other type of investor buys and sells at any moment at the price which is available in the market. Price fluctuations arise naturally as a result of changes in the balance of orders in the order book. At each time step a new order is submitted to the market. With equal probabilities this order can be a limit order to sell, a market order to sell, a limit order to buy, or a market order to buy. All orders are of the same unit size, and a new limit order to sell (buy) is placed with a random offset above (below) the most recent transaction price. The long memory of individual entries in this book gives rise to fat-tailed price distributions and volatility clustering. The Maslov model of stock market fluctuations was solved in the mean-field approximation in Ref. [20]. Statistical properties of an order book and the effect they have on price dynamics were studied in Refs. [19,21]. It was observed that the size distribution of market orders and limit orders (transaction sizes) has power law tails with an exponent close to 2. Also, it was found that a large imbalance in the number of limit orders placed at bid and ask sides of the book was shown to lead to a short term deterministic price change, which is in accord with the law of supply and demand.

Based on these insights, we will develop a model of stock price evolution, which would exhibit statistical properties very close to the empirical findings [22–25]. The proposed model will be a microscopic stock market model, similar to the Maslov model, but will have a different mechanism for the formation of the price. We will show that volatility is composed of the approximately constant part, traders' regret–risk aversion, and the time varying part, traders' uncertainty in expected return estimation.

2. Model

Let p_t be the price of a given stock. We define the stock price return as $r_t = \ln \frac{p_t}{p_{t-\Delta t}}$, where Δt is a given time interval. Without loss of generality we will focus on a one-period (one time step) case. A multi-period case will be considered later. In one-period the stock price at time $t = 0$ is p_0 and all market participants know that. Based on available information [5,17,26] they estimate the expected return at the time $t = \Delta t$ in an interval form $(\mu - \varepsilon, \mu + \varepsilon)$, where μ is the estimated expected return, and ε is the estimation uncertainty.

2.1. Configuration space

In order to define the space of elementary events Ω_N, we consider a company with M issued stock shares on a market with N agents (market participants). Every agent in every time step can buy or sell $\{0, \ldots, M\}$ shares. Thus, agent $i \in \{1, \ldots, N\}$ at every time step plays a strategy $\omega_i \in \{-M, \ldots, M\}$. This defines the configuration space as $\Omega_N = \{-M, \ldots, M\}^N$.

2.2. Personal preferences and interactions

Based on the assumptions, personal preferences of agents and interactions between them are postulated. For the sake of simplicity, we first focus on a case with no uncertainty, $\varepsilon = 0$. The difference between the supply and the demand is $\sum_{i=1}^{N} \omega_i$ [27]. Maslov and Mills studied empirically the average 1-min price change as a function of the initial imbalance of limit orders at the highest bid—lowest ask levels [19]. They found that the influence of the state of the order book on future prices is a real and sizeable effect. Furthermore, they stated that at their level of statistical errors it appears that the price change scales approximately linearly with the excess supply (or demand). Accordingly, the return is usually defined as [28,29],

$$r_N = \frac{1}{\lambda} \frac{\sum_{i=1}^{N} \omega_i}{N}, \tag{1}$$

where λ is the market depth, i.e., the excess demand needed to move the price by one unit. If we assume that all trading is done at the return r_N, the gain of agent i is

$$g_i = -\omega_i \left(\frac{1}{\lambda N} \sum_{i=1}^{N} \omega_i - \mu \right) \quad i = 1, \ldots, N. \tag{2}$$

For example, if agent i played strategy $\omega_i = 10$ (i.e. she wants to buy 10 shares) and if others played such strategies that $\frac{1}{\lambda N} \sum_{i=1}^{N} \omega_i > \mu$, then the agent has bought shares at a higher price than a fair one and has achieved a negative gain. A fair