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Physica A 370 (2006) 91–97
PHYSICA A
www.elsevier.com/locate/physa

Potential force observed in market dynamics

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Available online 11 May 2006

Abstract

As a model of market price, we introduce a new type of random walk in a moving potential, which is approximated by a quadratic function with its center given by the moving average of its own trace. The properties of resulting random walks are similar to those of ordinary random walks for large time scales; however, their short-time properties are approximated by abnormal diffusion with nontrivial exponents. A new data-analysis method based on this model enables us to observe temporal changes of potential forces from high-precision market data directly.

Keywords: Market prices; Potential force; Moving averages

1. Introduction

About 100 years ago Bachelier introduced a random walk model of market price [1] a little before the Einstein's famous paper on Brownian motion. It took nearly 70 years until his idea underwent a revaluation by the name of financial technology. In the formulation of financial technology the motion of market price is assumed to be a random walk without any market force. Although financial technology is now widely used in practical financial world, scientific validation is insufficient and deviations from real market data are pointed out [2,3].

In this paper we introduce a new market price model that is a random walk in a moving and deforming potential function. The center of the potential is given by a moving average of the walker's traces. Based on this model we can observe the change of potential function directly from market time series. The market data we analyze here is the high-precision data in yen–dollar exchange market consisting about 13 million bid prices in the period of 1995–2002.

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doi:10.1016/j.physa.2006.04.041

2. The moving averages

The first step of our analysis is to introduce the optimal moving average that makes it possible to separate a white noise from the raw data. For given time series of Yen–Dollar rate data, $\{\overline{p(t)}\}$, the moving average is defined by

$$\overline{p(t)} = \sum_{k=1}^{n} w_k p(t-k), \tag{1}$$

where the weights $\{w_k\}$ are determined so that the residual term $f(t)$ defined by the following equation becomes white noise:

$$p(t) = \overline{p(t)} + f(t). \tag{2}$$

It is generally possible to satisfy $|\langle f(t)f(t+T)\rangle| < 1.0 \times 10^{-2}$ for $T = 1$ to 1000 ticks with $n = 15$, where the bracket denotes the average over t, and time is measured by a pseudo-time called the "tick time," that is the count of transaction numbers [4]. In general the estimated weights $\{w_k\}$ for the yen–dollar rates are roughly approximated by an exponential function $w_k \propto \exp(-0.3k)$, namely, the characteristic decay time is about 3 ticks which corresponds to around 30 s in real time on average.

For the motion of this moving average we assume the existence of a kind of linear central force with its center given by another moving average which we call as "the super moving average." This operation is defined by a simple moving average of past M ticks of the optimal moving average as

$$\overline{P_M(t)} \equiv \frac{1}{M} \sum_{k=1}^{M} \overline{P(t-k)} \tag{3}$$

Fig. 1 shows an example of plots of raw market data, the optimal moving average and the super moving average. The value of M is typically from 8 to 64.

3. The potential force

The next step is to estimate the potential force by plotting the time difference of optimal moving average, $\overline{P(t+1)} - \overline{P(t)}$, versus the price difference between the optimal moving average and the super moving average at time t, $\overline{P(t)} - \overline{P_M(t)}$. If there is no force acting on the market as anticipated in financial technology, the plots should scatter around the horizontal axis. However, we can generally find nontrivial slope of this plot as typically shown in Fig. 2 and the slope is proportional to the strength of the market force. In order to observe this force we use 2000 data points.

As this force can be regarded as a central force, we can calculate the potential function by integrating the force along the horizontal axis in the preceding figure. Fig. 3(a) shows the examples of potential functions estimated for different values of M. The curvature or the strength of force is stronger for smaller M, and this

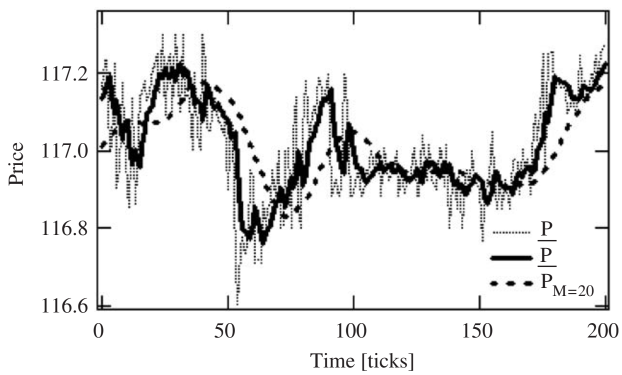

Fig. 1. An example of raw data (dots), the optimal moving average (line) and the super moving average with $M = 20$ (dash).