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Modeling stock market dynamics based on conservation principles

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Abstract

In this paper, a deterministic framework for modeling stock market dynamics is presented. The model is based on assets conservation principles and consists of a series of differential equations describing the dynamics of assets trading, and a (nonlinear) functional equation describing trade conservation (i.e., what is bought (sold) by one trader is sold (bought) by other traders). In this way, the dynamics of the assets and its price are determined by the trading dynamics. An equilibrium price is achieved when certain demand/supply equations are satisfied. Attention is devoted to a specific case, in which the trading activity is based on trader groups and an infinitely divisible asset. Numerical simulations show that even a single stock market asset with two classes of investors can display oscillatory price dynamics and instability. Moreover, the underlying oscillatory time-series display a discontinuous erratic-type behavior. © 2001 Published by Elsevier Science B.V.

Keywords: Econophysics; Stock market; Trading; Price dynamics; Oscillations

1. Introduction

Since the highly original work by Bachelier [1], following with the deep remarks by Working [2,3], the contributions by Samuelson [4], and the subsequent work of several economists and scientists, it has been arrived to the conviction that future asset prices tend to be largely random. This belief directed many investigations to seek theories to describe and understand such phenomenon. The underlying idea is that the evolution of

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the price of even a single stock market asset is commonly considered as an extremely difficult object of study. To interpret the price time-series from given information and to predict future values is such a discouraging and hopeless task, that the universally accepted approach is currently of probabilistic/statistical nature [4]. During the past 20 years, a very large literature developed concerning the statistical distribution of the changes in future prices (see for instances, Refs. [5,6]). In the physics literature, the statistical approach gave rise to an avalanche of publications demonstrating the application of mechanical statistical tools to explain regularities in e.g., price volatility and fundamentals [6]. Recent issues of *Physica A* showcase the application of physics methodologies to understand the dynamics of stock markets. An example of application of stochastic analysis with a physics viewpoint is the description of price changes in open markets [7–9].

The major problem with the statistical approach is that it is incapable of understanding the underlying dynamics governing price evolution. For instance, it is not clear how the heterogeneity and composition of agents in a stock market affect the price dynamics [10]. Moreover, arguments based on the deepest statistical machinery seems to erode seriously the confidence of the famous Efficient Market Hypothesis and its sequel [11]: the random walk model for prices of financial assets. This state of events has led some researchers to take alternative approaches: Is there a nonlinear methodology (e.g., deterministic models) as an alternative to the stochastic approach, which generates a time-series sequence of price changes that appear random when in fact such sequence is nonrandom? The issue of deterministic chaos to explain price volatility has been documented (see for instance, Refs. [12,13]). The empirical analysis on price time-series presented in the book of De Grauwe et al. was not conclusive of chaos in asset markets. Specifically, in no case could they find a strange attractor. Major criticisms have been made to the deterministic chaos models [10]. If a model is chaotic, then short run forecasting is quite feasible, but longer run forecasting is impossible. The empirical evidence in asset price changes is just the opposite. For instance, short period (e.g., monthly or quarterly) rates of inflation cannot be predicted, but in the long run period the quantity theory of money is valid [14]. Malliaris and Stein [10] have suggested that price volatility processes reflect the output of a higher-order dynamic system with an underlying stochastic foundation, and analyzed the economic scenarios which may generate seemingly chaotic processes that can be interpreted statistically. They constructed an intertemporal price determination model to explain the learning process and the efficient use of information. Based on such model, analysis on actual financial market (S& P 500) and agricultural commodities, it has been concluded that the pure random walk hypothesis of price changes is rejected. Although Malliaris and Stein's model was archetype, it can be considered as a first attempt to systematize and clarify the underlying mechanisms determining price volatility.

In the spirit of Malliaris and Stein's approach, a deterministic framework for modeling stock market dynamics is presented in this paper. The model is based on assets conservation principles to reflect asset trading among different traders classes. The main goal of our modeling approach is to gain insight and understanding of the