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Fractional market dynamics

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Abstract

A new extension of a fractality concept in financial mathematics has been developed. We have introduced a new fractional Langevin-type stochastic differential equation that differs from the standard Langevin equation: (i) by replacing the first-order derivative with respect to time by the fractional derivative of order μ; and (ii) by replacing "white noise" Gaussian stochastic force by the generalized "shot noise", each pulse of which has a random amplitude with the α-stable Lévy distribution. As an application of the developed fractional non-Gaussian dynamical approach the expression for the probability distribution function (pdf) of the returns has been established. It is shown that the obtained fractional pdf fits well the central part and the tails of the empirical distribution of S&P 500 returns. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Dynamics of financial assets demonstrate the stochastic behavior. The first theoretical attempt to describe stochastic financial dynamics was made by Bachelier in 1900 [1]. He proposed the Brownian motion to model the stochastic process of the return $G(t) \equiv G_{\Delta t}(t)$ over a time scale Δt defined as the forward change in the logarithm of price or market index $S(t)$,

$$G_{\Delta t}(t) = \ln S(t + \Delta t) - \ln S(t) .$$

Bachelier's approach is natural if one considers the return over a time scale Δt to be the result of many independent "shocks", which then lead by the central limit theorem to a Gaussian distribution of returns [1]. The Gaussian assumption for the

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dynamics of a financial assets is widely used in mathematical finance because of the simplifications it provides in analytical calculation; indeed, it is the main assumption used in the famous Black–Scholes option pricing formula [2].

However, empirical studies [3–5] showed that the probability distribution of returns has pronounced tails in striking contrast to that of a Gaussian.

Mandelbrot, who introduced into scientists' lexicon the new term "fractal", observed [3] that in addition to being non-Gaussian, the stochastic process of returns shows another interesting property: self-similarity – that is, the statistical dependencies of returns have similar form for various time increments Δt, ranging from 1 d to 1 month. As it is discussed in Ref. [6] "motivated by (i) the pronounced tails, and (ii) the stable functional form for different time scales, Mandelbrot [3] proposed that the distributions of the returns is consistent with a Lévy stable distribution [7] – that is, the returns can be modeled as a Lévy α-stable process". Thus, from the point of view of the fractal concept one may say that Bachelier's and Mandelbrot's approaches were the first attempts applying the fractality concept to model the financial assets dynamics. It is well known that the trajectories of the Brownian and Lévy stochastic processes are fractals. It means that they are non-differentiable, self-similar curves whose fractal dimensions are different from their topological dimension [8].

Since the well-known papers [3,9] on Lévy distributions, there have been several attempts to develop the fractional approach to the problem. Most of them deal with cut-off of the Lévy distributions (see, for example, Refs. [10,11]). The approaches based on cut-off procedures are approximations to the pdf trying to fit the empirical data, but they are essentially non-dynamical and do not allow one to predict the future behavior of a market.

We develop a new extension of a fractality concept in financial mathematics and apply it to describe the stochastic dynamics of the stock and currency markets. We propose a new fractional dynamical approach to model the evolution of the stochastic financial assets. The main difference from the previous stochastic dynamical approaches to fluctuating market phenomena is the following. We consider the fractional Langevin-type stochastic differential equation that differs from the standard Langevin equation:

(i) By replacing the first derivative with respect to time by the fractional derivative of order μ.
(ii) By replacing the "white noise" Gaussian stochastic force by the generalized "shot noise", each pulse of which has a random amplitude.

The proposed fractional dynamical stochastic approach allows to obtain the probability distribution function (pdf) of the modeled financial asset. As an application of the developed general approach, we derive the equation for the pdf of increments Δx of a financial market index as a function of the time delay Δt, $\Delta x(\Delta t) = x(t + \Delta t) - x(t)$, where the value of the index is denoted as $x(t)$. Statistical properties of asset price increments play an important role both for understanding of the markets dynamics and for financial engineering applications, for instance, the pricing of derivative products and risk evaluations. The theoretically predicted pdf of increments of market index Δx

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