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Optimal length of lease period and maintenance policy for leased equipment with a control-limit on age

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ABSTRACT

This paper investigates the optimal length of lease period and maintenance policy for leased equipment. In order to have steady revenue, the lessor (owner) of equipment may provide a discount to encourage the lessee (user) to sign a contract with a longer lease period. However, as the lease period increases, the maintenance cost also increases due to the deterioration or usage of the equipment. Therefore, there is a need to determine an appropriate length of lease period and a corresponding maintenance policy for the leased equipment from the viewpoint of the lessor such that the expected total profit is maximized. In this paper, the following maintenance scheme is considered. When the leased equipment fails, a minimal repair is conducted to bring the equipment back to an operating condition. Furthermore, when the age of the equipment reaches a specified level (called as a control-limit), an imperfect preventive maintenance (PM) is carried out to avoid possible failures. Under this maintenance scheme, the mathematical model of the expected total profit is developed and the optimal length of lease period and the corresponding optimal maintenance policy are derived. Finally, some numerical examples are given to illustrate the effects of the length of lease period and the maintenance policy on the expected total profit.

1. Introduction

Due to consumers' various requirements on the functionality of a product and the rising price of some production equipment, it may not be economical for a manufacturer to purchase expensive production equipment to produce the multi-functional products requested by consumers. Instead, manufacturers may lease these production equipments rather than purchasing them [1]. In general, the content of a lease contract usually includes the lease period, rent, new/old equipment, preventive maintenance (PM) schedule, tolerable time of repair, and penalty. Especially, the length of lease period and the maintenance and service of production equipment usually play a critical role in the contract.

For repairable leased equipment, the maintenance actions can be classified into two major categories, (i) Corrective Maintenance (CM) and (ii) Preventive Maintenance (PM). CM actions are used to rectify failed equipment back to its operational state, and PM actions are performed to improve the operational state of the equipment to avoid failures. For CM action, minimal repair is often adopted to restore failed equipment [2]. After minimal repair, the equipment is in normal operation but the failure rate function remains unchanged. As to preventive maintenance, it can be further classified into perfect PM actions and imperfect PM actions. Perfect PM restores the equipment to its original (new) condition and imperfect

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PM brings the equipment back to somewhere between its original and current condition. Perfect or imperfect PM is often adopted in practice to reduce the number of equipment failures. Various PM policies have been proposed and studied for different situations, such as finite or infinite horizon [3–6], and periodic or sequential [7–10].

For the imperfect PM of equipment, two methods have been proposed [11]: (i) age-reduction method (ARM), in which the age of the equipment is restored to younger than the current age after each PM action, and (ii) failure rate reduction method (FRRM), in which the failure rate of equipment is reduced after each PM action. Yeh and Lo [12] used ARM to derive the optimal periodical and sequential PM warranty policies for repairable products. Wang [13] summarizes, classifies, and compares various existing maintenance policies (for example, age-dependent PM policy, period and sequential PM policies, failure and repair limit policies et al.) for both single-unit and multi-unit systems. Wu and Zuo [14] formulated the expected costs of the linear, nonlinear, and hybrid PM models and derived the properties of the PM models. On the other hand, Pongpech and Murthy [15] utilized FRRM to derive the optimal periodical maintenance policy for leased equipment. Jaturonnatee et al. [16] derived the optimal number and degrees of PM for leased equipment incorporated with corrective minimal repairs. Yeh and Chang [17] investigate the threshold value of failure rate for leased products with a Weibull lifetime distribution and obtain the optimal threshold value and optimal maintenance policy within the lease period. Most studies mentioned above focus on determining the optimal PM policy for leased equipment with a specified leased period. However, the length of leased period may be a critical issue in signing a lease contract. Hence, this paper will investigate the length of lease period by using ARM to describe the degree of PM and construct a mathematical model of the expected total profit within the lease period. Furthermore, the optimal length of lease period and the corresponding optimal maintenance policy are derived such that the expected total profit is maximized.

This paper is organized as follows. The expected total profit model of leased equipment within the lease period is developed in Section 2. The properties of the optimal maintenance policy and number of lease periods are investigated in Section 3. In Section 4, the effect of the maintenance policy and the number of lease periods on the expected total profit is illustrated through numerical examples. Finally, some conclusions are drawn in Section 5.

2. Mathematical formulation

To construct the mathematical model, the following notations are adopted in this paper:

L unit length of lease period

k number of periods leased when each period of the equipment is L

H the payment for the equipment in the first lease period

$f(t)$ probability density function of the lifetime of the leased equipment

$r(t)$ failure rate function of the leased equipment

$R(t)$ cumulative failure rate function of the leased equipment

C_m minimal repair cost

C_f penalty cost for each failure

θ_a control-limit of age for performing PM actions

x maintenance degree of a PM action

$C_p(x)$ PM cost function with maintenance degree x

n total number of PM actions within the lease period

t_d life cycle of the equipment

V the purchase price of the equipment

V_d residual value of equipment at time t_d

ϕ discount rate of lease payment($0 \leq \phi \leq 1$)

$E[TC]$ expected disbursement within the lease period

$E[TR]$ expected income within the lease period

$E[TP]$ expected profit within the lease period

Consider that a new equipment is leased with the total lease period kL. Within the lease period, any equipment failure is corrected by a minimal repair. A minimal repair is performed with a fixed cost $C_m > 0$. There is a penalty cost $C_f > 0$ for the lessor when the leased equipment fails. Within the lease period, when the age of the equipment reaches a certain control-limit θ_a, the imperfect PM actions should be carried out with maintenance degree x. After an imperfect PM action is performed, the age of the equipment becomes x unit of time younger than before and each PM incurs a cost of $C_p(x)$. In practice, the PM cost is a non-negative and increasing function of maintenance degree x.

For repairable leased equipment, consider that the failure rate $r(t)$ is an increasing function with $r(0) = 0$. Suppose that the minimal repair time and the time required for performing an imperfect PM action are negligible, the maintenance scheme can be described in terms of the failure rate function as shown in Fig. 1. As a result, when minimal repairs are