



# Optimal imperfect maintenance strategy for leased equipment



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## ABSTRACT

This paper presents a preventive maintenance (PM) policy for leased equipment over a finite lease period  $L$ . Maintenance is entrusted to the lessor. The purpose is to determine the PM period  $T$  minimizing the expected total cost over the lease period in a context where repairs and PM actions are imperfect and have non-negligible durations. The leased equipment is periodically subjected to imperfect PM restoring it to the state 'as good as new' with probability  $p$  and keeping it at state 'as bad as old' with probability  $q$ . In case of failures between successive PM actions, imperfect repairs are performed with a given efficiency following a decreasing quasi-renewal process. Besides the maintenance costs, the lessor may incur a penalty cost if the total expected equipment downtime due to repair and PM during the lease period exceeds a pre-specified threshold. A mathematical model and a numerical algorithm are developed to find the optimal maintenance strategy  $T^*$  over the lease period for any given set of input parameters (equipment reliability, the lease period length, the repair and PM efficiency levels, and the maintenance and penalty costs). A numerical example is presented and the obtained results are discussed.

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## 1. Introduction

Leasing has become a widely used business strategy. A business model in which a company leases new products and sells re-manufactured products at the same time has been presented by Aras et al. (2011). The leasing option is getting more and more attractive especially when dealing with expensive industrial, medical, transportation, and other types of complex equipment. It is also frequent for such equipment that leasing contracts stipulate that maintenance of the leased equipment is entrusted to the lessor. This is generally the case when maintenance requires expensive tools and highly skilled labor having a specific know-how. In such situations, the lessor has to establish an effective maintenance strategy to be specified in the lease contract. Such a strategy will generally include preventive maintenance in order to reduce the number of potential failures and related penalties during the lease period (Wu et al., 2011).

Several issues related to preventive maintenance (PM) policies for leased equipment have been discussed in the literature during the last decade. Jaturonnatee et al. (2006) modeled a sequential PM strategy over the lease period involving minimal repair at failure and imperfect PM actions. They sought to determine the

optimal PM policy (the number, the instants and the level of PM actions) that minimizes the total expected cost incurred by the lessor. The effect of PM actions is modeled by a reduction of the failure intensity. Pongpech and Murthy (2006) developed a mathematical model for a periodic PM policy to derive the optimal PM period and the optimal reduction in the intensity function at each PM such that the total expected cost is minimized. They assume that the lessor incurs penalty costs for each failure that occurs over the lease period and also if the time to repair a failure exceeds a specified time limit. Their model has been compared with the one proposed by Jaturonnatee et al. (2006). They proved that their proposed policy can be easier to implement in practice. Pongpech et al. (2006) considered leased second-hand equipment subjected to a number of imperfect PM actions and minimal repairs following failures over the lease period. PM actions reduce the rate of occurrence of failures. The authors developed a model to derive the optimal upgrade of the used equipment, the optimal number of PM actions, the optimal PM instants and the optimal PM efficiency levels which minimize the total expected cost over the lease period including maintenance and penalty costs incurred by the lessor. They consider a penalty due to the exceeding of a repair time limit and a penalty due to the exceeding of a specified number of failures over the lease period. They investigated the impact of these penalties on the optimal maintenance strategy. Yeh et al. (2009) developed a mathematical model and an algorithm to derive simultaneously the number of PM actions to be performed, the time intervals between them and the optimal PM

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Notation	
$L$	lease period
$T$	PM period (decision variable)
$n$	number of PM actions to be carried out over the lease period $L$
$F_{T_1}$	probability distribution function associated with the first failure instant
$T_p$	PM action duration, a constant
$T_c$	repair duration, a constant
$a$	repair efficiency factor
$X$	time interval between the last preventive action and the end of the lease period $L$ $X=L-n(T+T_p)$
$m$	number of scenarios
$c_p$	cost of a PM action
$c_c$	cost of a repair
$c_a$	penalty cost per time unit
$C_{PM}$	total cost of PM during the lease period $L$
$C_R$	expected total cost of repair during the lease period $L$
$C_P$	expected total penalty cost
$C(T)$	expected total cost incurred during the lease period $L$
$C_j(T)$	expected total cost incurred for scenario $S_j$ ( $j=1, \dots, m$ )
$P_{S_j}$	probability of occurrence of scenario $S_j$
$N_{r_j}$	expected number of system restarts (failure-repair) during the lease period $L$ in the case of scenario $S_j$
$N_r$	expected number of system restarts (failure-repair) during the lease period $L$
$Q(t)$	the quasi-renewal function which corresponds to the expected number of system restarts (failure-repair) within the time interval $[0, t]$
$Q_i$	expected number of system restarts until instant $t$ ( $i=1, \dots, n$ ); $Q_0=0$
$D_T$	total expected equipment downtime due to repair and PM during the lease period $L$
$D_{T_j}$	expected equipment downtime due to repair and PM in case of scenario $S_j$ ( $j=1, \dots, m$ )
$D_{T_{max}}$	maximum downtime allowed before a penalty has to be paid
$\lfloor Z \rfloor$	integer part of a real number $Z$

efficiency level such as the expected total cost is minimized. They considered a fixed failure rate reduction for all PM actions and minimal repairs following failures during the lease period. A closed-form solution for equipment whose lifetime follows a Weibull probability distribution has been obtained. Their optimal PM policy has been compared with two other policies of [Jaturonnatee et al. \(2006\)](#) and [Pongpech and Murthy \(2006\)](#). They proved that their maintenance strategy can be either equivalent or more effective than the two other ones. The same problem has been addressed by [Yeh et al. \(2011\)](#) considering PM actions inducing a fixed age reduction and minimal repair at failure. They derive the optimal PM policy minimizing the expected total cost incurred during the lease period. They suppose that the equipment time to failure follows a Weibull probability distribution and they consider a linear relationship between the PM cost and its efficiency level. They also take into account a repair time limit penalty as part of the total cost incurred by the lessor. More recently, [Zhou et al. \(2014\)](#) proposed a multi-phase preventive maintenance policy for leased equipment. The effect of PM is modeled by an age reduction method and failures are minimally repaired. The aim is to minimize the cumulative maintenance and penalty cost throughout the lease period. [Ben Mabrouk et al. \(2016\)](#) tackled the problem considering the equipment over its whole lifecycle made of several successive lease periods. They propose a mathematical model to determine the optimal efficiency levels of PM actions to be performed on the equipment between successive lease periods so as to maximize the total expected profit of the lessor over the equipment lifecycle. They considered the same warranty duration for each lease period during which failures are minimally repaired by the lessor. They proposed a genetic algorithm to solve complex instances of the problem involving several lease periods and different possible PM efficiency levels. PM policies for leased equipment have also been discussed in other papers (see for instance [Yeh and Chang, 2007](#); [Chang and Lo, 2011](#); [Schutz and Rezg, 2013](#)).

All the above cited papers on the development and optimization of PM policies for leased equipment have in common the fact of considering imperfect PM actions and minimal repair at failure. The latter assumption of minimal repair has the advantage of making the computations of the average repair cost tractable. However, in many situations in practice, restoring failed equipment through minimal repair may not be relevant. Indeed, corrective maintenance following failure does not necessarily keep

the equipment failure rate at the same level as it was before failure (as bad as old). It is generally an imperfect repair restoring the system to a state between 'as bad as old' and 'as good as new'. A classification of types of repairs according to the depth of repair can be found in [Chukova et al. \(2004\)](#). Besides, all of the existing models suppose that PM actions have negligible durations. This is not necessarily true in practice. PM actions could last a relatively long time in many situations particularly for complex equipment, inducing a downtime that can be penalizing particularly if it exceeds a given threshold judged as reasonable by both parties when establishing the leasing contract.

In this paper, we address the same problem of maintenance strategies for leased equipment, but contrarily to previous published works, we consider PM and repair actions as being both imperfect and having non negligible durations. Within this context which is more relevant from a practical point of view, we propose a maintenance strategy involving imperfect repairs and PM. Imperfect PM actions are performed periodically (every  $T$  time units) during the lease period. They follow a  $(p, q)$  rule, which means that they restore the equipment to the 'as good as new' state with probability  $p$  and they keep it in the 'as bad as old' state with probability  $q$  ( $q=1-p$ ). As for imperfect repairs, they follow a decreasing quasi-renewal process.

The quasi-renewal process was introduced by [Wang and Pham \(1996, 1997\)](#). Several other works followed using this modeling process of imperfect repairs (see for example: [Rehmet and Nachlas \(2009\)](#), [Samet et al. \(2010, 2012\)](#), [Park and Pham \(2010\)](#)). Quasi-renewal has proved to be an appropriate modeling approach of imperfect repairs especially for situations where repairs do not change the failure mechanism of the equipment. For example, it has been effectively used in the case of gear boxes maintenance in Kowloon Motor Bus Company, in Hong Kong ([Leung and Fong, 2000](#)). It has also been applied in the case of industrial refrigerators in a Turkish beverage company ([Samatli-Paç and Taner, 2009](#)).

To the best of our knowledge, in the works dealing with imperfect maintenance strategies based on the quasi-renewal approach, cost models have never been expressed over a finite horizon which is of interest in this work (finite leasing period). It has been clearly established that it is more difficult to model optimal maintenance policies over a finite time span than over an infinite time span (see [Nakagawa and Mizutani, 2009](#)).

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