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The freight allocation problem with all-units quantity-based discount: A heuristic algorithm

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ABSTRACT

This paper studies a problem encountered by a buying office for one of the largest retail distributors in the world. An important task for the buying office is to plan the distribution of goods from Asia to various destinations across Europe. The goods are transported along shipping lanes by shipping companies, which offer different discount rates depending on the freight quantity. To increase the reliability of transportation, the shipper imposes a quantity limit on each shipping company on each shipping lane. To guarantee a minimum business volume, each shipping company requests a minimum total freight quantity over all lanes if it is contracted. The task involves allocating projected demand of each shipping lane to shipping companies subject to the above conditions such that the total cost is minimized.

Existing work on this and related problems employs commercial linear programming software to solve their models. However, since the problem is $\mathcal{NP}-$hard in the strong sense, it is unlikely to be solvable optimally in reasonable time for large cases. Hence, we propose the first heuristic-based algorithm for the problem, which combines a filter-and-fan search scheme with a tabu search mechanism. Experiments on randomly generated test instances show that as the size of the problem increases, our algorithm produces superior solutions in less time compared to a leading mixed-integer programming solver.

1. Introduction

This study is motivated by a project awarded by a Hong Kong-based buying office (henceforth referred to as the *shipper*) for one of the largest international retail corporations in the world with over 2000 outlets in Europe, Africa and Asia. The shipper annually procures diverse products, from textiles and food stuffs to major electrical appliances, from over one thousand suppliers across Asia to satisfy the demands of parent company's sales divisions that are distributed across Europe. Long-distance ocean shipping is the main transportation mode for the shipper for the delivery of the procured products, accounting for around 95% of its total annual turnover on average. Hence, the shipper maintains close relationships with most of the leading international shipping companies (henceforth referred to as the *carriers*), allowing the shipper to enter into long-term contracts with these carriers at discounted rates. The transportation network under the purview of the shipper comprises more than 1000 shipping lanes connecting 71 loading ports in Asia and 27 discharging ports in Europe. This paper examines the problem of allocating the freight quantity for all lanes to the carriers such that the total transportation cost is minimized.

The shipper performs this freight allocation at the strategic level. At the beginning of every fiscal year, the shipper forecasts the total quantity of freight (demand) for the coming year on each lane, taking into account possible market fluctuations. Price quotes are collected from each carrier along with its discount information and minimum quantity commitment (MQC) [1]. The carrier offers discount rates to the shipper according to the total freight quantity it obtains across all lanes, and if it is contracted this total freight quantity must exceed its requested MQC. As a safeguard against the inability of a carrier to fulfill its contractual obligations due to unforeseen circumstances, the shipper also specifies a minimum number of carriers for each lane.

We call this problem the freight allocation problem with all-units quantity-based discount (FAPAQD). While some research has been done on problems of this nature, in general they employ exact linear and integer programming solvers to produce optimal solutions on small instances. However, the problem is $\mathcal{NP}-$hard in the strong sense, and therefore such approaches are unlikely to be successful for the large and practical scenarios faced by the shipper. Consequently, we developed the first tailored heuristic for the FAPAQD, which makes use of the polynomial-time

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solvable min-cost network flow problem to generate solutions, and then uses a filter-and-fan search heuristic with tabu search to locate good solutions.

In our problem, the discount rates offered by each carrier are expressed as *discount intervals*. If the discount interval adopted for each carrier is determined, the resultant model can be solved in polynomial-time. Thus, identifying the best discount interval to select for each carrier would allow us to find the optimal solution to this problem. Our approach is based on searching the space of combinations of discount intervals for the carriers. In this paper, we propose a filter-and-fan technique with tabu search (F&F) for this purpose; computational experiments on randomly generated instances of practical size show that our approach outperforms CPLEX 11.0 in terms of both solution quality and computation time. Note that other than the F&F, we have also investigated using a standard simulated annealing algorithm or tabu search algorithm with similar neighborhood structures and a variety of parameter settings, but preliminary experiments indicate that our F&F outperforms both these approaches.

The rest of the paper is organized as follows. In Section 2, we provide an overview of existing research that considers the MQC constraint or discounts. We then describe the FAPAQD in detail in Section 3, where we explain the notations used in the remainder of the paper and formulate the problem as a mixed-integer programming (*MIP*) model. This is followed in Section 4 with a description of the F&F for this problem. Our experimental results are given in Section 5, and we conclude our paper in Section 6 with some closing remarks.

2. Literature review

The MQC constraint has been previously studied in the analysis of supply contracts [2–4]. In c*ommitment-purchase contracts*, buyers commit in advance to purchasing a minimum quantity of products from the supplier, and the unit price of the product is based on the total quantity or total dollar amount purchased. In recent years, the transportation service procurement problem with the MQC constraint has also received attention. Lim et al. [1] incorporated the MQC constraint into the traditional transportation problem, which makes the problem intractable. The authors proposed an *MIP* model defined by a number of strong facets and applied a branch-and-cut scheme, a linear programming rounding heuristic, and a greedy approximation method. This work was extended in Lim et al. [5] by considering a fixed selection cost associated with each carrier; for this extended problem it was shown that not only is finding an optimum solution NP-hard in the strong sense, but finding a feasible solution is also $\mathcal{NP}-$hard. The problem studied in our paper can be regarded as another extension of Lim et al. [1].

There is a substantial amount of literature related to procurement problems involving discounts. Two commonly used discount policies are the *all-units discount* and the *incremental discount*. Under the all-units discount policy, the discounted price applies to all units purchased, while the discounted price applies only for quantities within the associated discount level for the incremental discount policy. The discount can be based on *total business volume*, which is the total dollar amount of business across all products purchased from the suppliers [6]; or based on *total quantity*, in which the discount is given according to the total number of units of all products purchased from the suppliers [7]. If these problems are distinguished by the number of suppliers and products, then procurement problems with discounts can be classified into the following four categories: (1) single supplier and single product [8–11]; (2) single supplier and multiple products [12–17]; (3) multiple suppliers and single product [18–21]; and (4) multiple suppliers and multiple products [6,7,22–26]. The FAPAQD can be viewed as a multiple-supplier, multiple product procurement problem; the rest of this section provides an overview of existing work of this type.

Katz et al. [22] and Sadrian and Yoon [6] introduced a Procurement Decision Support System (PDSS) that improved the cost-effectiveness of purchasing activities of regional Bell telephone companies. The PDSS uses the optimization software LINGO, but the detailed numerical results were not revealed by the authors. The model in Crama et al. [23] considers a company that manufactures a set of products, each of which can be obtained by blending a set of ingredients according to certain recipes; ingredients are purchased from a number of suppliers who offer discount schedules. This model is more complex than the one examined in our study because it requires the concurrent determination of the recipe used for each product along with the quantity of each ingredient purchased from each supplier. Small test instances for this problem were solved by a branch-and-bound algorithm embedded in the XA solver.

Xia and Wu [24] formulated the procurement problem as a multi-objective *MIP* model and utilized the optimization toolbox in MATLAB to solve the problem. Stadtler [25] presented a general model that is applicable to both all-units and incremental discount policies, and solved the model using the standard *MIP* solver Xpress-MP optimizer. Goossens et al. [7] proposed a min-cost network flow based branch-and-bound algorithm that uses the commercial *MIP* solver CPLEX 8.1 to solve the procurement problem under a total quantity discount structure optimally. However, the experimental results show that this algorithm is only applicable to small instances as its performance is worse than CPLEX 8.1 when solving medium to large instances. Sawik [26] studied procurement models that simultaneously consider discount and some other influence factors, such as price, quality of purchased parts and reliability of on time delivery. The author conducted the experiments using the AMPL programming language and the CPLEX 11.0 solver with the default settings.

Importantly, the test data employed for the evaluation of the above approaches are all much smaller than the hundreds of lanes and dozens of carriers that our problem must consider; for example, the largest instances considered by Goossens et al. [7] consist of only 50 suppliers and 100 products. While problem instances of this scale are appropriate for the procurement problems examined in these publications, they are insufficient for our purposes.

3. Problem formulation

We modeled the problem faced by the shipper in the following manner. There is a set of candidate carriers $I=\{1, 2,\dots, n\}$ and a set of lanes $J=\{1, 2,\dots, m\}$. Not all carriers can operate on all lanes; the set N contains (i, j) pairs, $i\in I$, $j\in J$, indicating that carrier i operates on lane j. The projected demand for lane j in the upcoming fiscal year is given by d_j.

Each carrier i has an MQC, denoted by b_i, which defines the minimum quantity that must be assigned to that carrier if it is selected. The regular price quoted by carrier i to transport one unit of product on lane j is denoted by p_{ij}. Each carrier i also defines a set of *discount intervals* $K_i=\{1, 2,\dots,k_i\}$ that describe the percentage discount on all units assigned to that carrier for each quantity range. Table 1 shows examples of discount intervals for two carriers along with their corresponding MQCs, where both carriers have defined 4 discount intervals. We denote the discount lower bound of the kth interval for carrier i by β_{ik}, and the discount value of the kth interval for carrier i by a_{ik}. In the example, the values for Carrier 1 are $\beta_{11}=2400$, $\beta_{12}=3000$,