Insurance: Mathematics and Economics 52 (2013) 35–45

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

# If we can simulate it, we can insure it: An application to longevity risk management[☆]

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ARTICLE INFO

*Article history:*
Received April 2012
Received in revised form
October 2012
Accepted 23 October 2012

*JEL classification:*
G22
G23

*Keywords:*
Least squares Monte Carlo
Longevity risk
Reinsurance
Simulation

ABSTRACT

This paper proposes a unified framework for measuring and managing longevity risk. Specifically, we develop a flexible framework for valuing survivor derivatives like forwards, and swaps, as well as options both of European and American style. Our framework is essentially independent of the assumed underlying dynamics and the choice of method for risk neutralization and relies only on the ability to simulate from the risk neutral process. We provide an application to derivatives on the survivor index when the underlying dynamics are from a Lee–Carter model. Our results show that taking the optionality into consideration is important from a pricing perspective.



## 1. Introduction

In 1986 the rock band *Queen* sang "Who wants to live forever?" in which they conclude "Who waits forever anyway?" They could have easily put in song "Who wants to live old and poor forever?" for that is the risk of outliving our assets. Indeed, outliving our assets is not an enviable outcome of increased longevity at older ages, and anyone would probably try to avoid running out of funds when they are old and grey. Sadly, however, lacking financial resources is one of the pernicious effects stemming from the risk of a population living longer than planned, so that not enough money has been saved to fulfill its financial needs. We refer to this risk as longevity risk. Put differently, longevity risk arises when the annuitants' actual life expectancy is greater than their expected life expectancy. Unknown longevity affects the overall profitability of institutions that offer lifetime pensions, such as large corporations and governments, as well as the total savings of individuals.

Financially, aggregate longevity risk is mostly negative since retirement systems in developed countries rely on aggregate mortality rate forecasts to calculate an individual's pension benefits until his death (not to mention all other costs associated with old-age, such as medicare and disability benefits). When the life expectancy of a cohort of pensioners exceeds forecasts, the retirement system must pay this cohort income that exceeds the level initially projected and accumulated. It is then exposed to the risk of lacking the capital needed to meet its financial commitments. The problem is similar for individuals who are accumulating wealth in expectation of converting all that wealth into annuities upon retirement (see Levhari and Mirman, 1977, and Davies, 1981); an unanticipated increase in longevity will lead to insufficient wealth accumulation on the part of an individual who will then need to either reduce consumption more than anticipated or delay retirement altogether (see Cocco and Gomes, 2012).[1]

Considering the size of the risk exposure, the only way to potentially manage longevity risk is, arguably, by drawing upon the wealth of the entire capital market. The argument is essentially that even though potential losses due to longevity risk are larger

☆ The authors thank Joanna Mejza for fascinating discussions and Amélie Favaro for excellent research assistance. This research is financially supported by the Social Science and Humanities Research Council of Canada as well as by CIRANO.

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[1] For a detailed discussion of this annuity puzzle, see Brown (2004). The puzzle is derived from the mismatch between annuity purchases and what would have been optimal under a lifetime consumption model à la Yaari (1964). For more on this topic, see the recent Huang et al. (2012) paper.

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doi:10.1016/j.insmatheco.2012.10.003

than what any one insurer or group of insurers can assume, such a loss exposure is not uncommon for the capital markets. Put differently, what is a systemic risk for the life insurance market is an idiosyncratic risk for the entire capital market. The ability of capital markets to absorb larger amounts of risk than the market for life reinsurance stems from two factors. First, capital markets are very large and much larger than the market capitalization and surplus of the insurance industry, and second, there is virtually no correlation between longevity risk and market risk so that exposure to longevity risk shifts the portfolio frontier up and to the left. The benefit to capital market participants of assuming this type of risk is that it gives money managers access to a new class of assets. Because of the negligible correlation between longevity risk assets and the money managers' portfolio of financial assets, having exposure to longevity risk can decrease the volatility of their entire book of business.

Capital markets have started to develop products that allow financial institutions to invest in mortality and longevity. The primary example of such a product is the mortality swap whereby one party will pay a measure of expected mortality and in return will receive a measure of actual mortality experience (for a discussion of other types of longevity derivatives see Blake et al., 2006, *inter alia*). One of the reasons why capital markets have been relatively slow in developing longevity products is that calculating their fair value raises many challenges. First of all, mortality, unlike financial assets, is not a traded asset and therefore longevity products cannot be valued simply by the absence of arbitrage (under the risk-neutral measure) since no replication portfolio can be built. It is therefore necessary to use incomplete market valuation methods that imply the existence of a market risk premium, which can only be appraised using market data. Secondly, given that the cash flows of these products are directly linked to mortality or survival rates their valuation requires the ability to forecast these rates accurately. While this could be achieved using mortality projection models, these are oftentimes somewhat non-standard, at least from a derivatives pricing perspective.

The goal of this paper is to develop a flexible framework for pricing essentially all types of longevity derivatives which is independent of the assumed dynamics. For this reason we choose to use simulation techniques. Simulation techniques are widely used in the finance literature to price derivatives. We show how this framework can be used to price survivor forwards, swaps, and both European and American style options. Though it is possible to obtain closed-form, and hence simpler, formulas for some longevity forwards and swaps, and for some European options, such closed-form solutions generally require restrictive assumptions about the dynamics of the underlying risk factors. For example, Dawson et al. (2010) rely on a tightly parameterized beta distribution to derive their closed-form Black–Scholes–Merton-type prices for the European swaption. Compared to this our framework is much more flexible and all that is needed is the ability to simulate from the risk neutral distribution, something that is often the case after an appropriate distortion or transformation.

Moreover, even if the choice of model allows for closed-form solutions for some types of derivative contracts, this will not be the case for derivatives that allow, e.g., early exercise (such as the American option) or whose value is path dependent (such as up-and-in and down-and-out options, or Asian options). Thus, in order to price these instruments numerical methods have to be used. One important advantage of the simulation method is that it can easily be augmented to take path dependence into consideration. It can also be extended to incorporate possible early exercise. Finally, it should be noted that simulation methods are particularly well suited to price derivatives on multiple underlying risks. In particular, the computational complexity of the simulation technique grows linearly in the number of risks, whereas most other numerical methods that exist for option pricing are plagued with a computational complexity that grows exponentially in the number of risks—and hence are said to suffer from the "curse of dimensionality".

The simulation framework we propose in the current paper is flexible enough to accommodate the particular features of the insurance mortality data. As an application, we show how it can be applied to manage longevity risk where the mortality distribution follows a Lee–Carter model, though any other model could be used (see Robine, 2011, and Gaille, 2010, for surveys of mortality models used in the current context).[2] To be specific, we price survivor forwards, swaps, and options in this unified framework. Our results show that taking the optionality into consideration is important from a pricing perspective and if this is neglected the value of these products is severely underestimated. Since insurance and reinsurance contracts can be seen as put options, this finding has non-trivial implications for the valuation of such contracts. In fact, the inherent flexibility of the simulation method we propose allows one to price efficiently various types of derivatives (such as bull and bear spreads, knock-in and lookback options, etc.) which, in turn, allows one to design and price products that enable insurers and reinsurers to manage risk exposure efficiently. Thus, a framework as flexible as the one we propose has extraordinary applications for the management of risk in the context of longevity risk.

The rest of this paper proceeds as follows. In Section 2 we provide a review of some of the products that have been proposed in the market for longevity risk. We also suggest a new product: a survivor option with time varying strike prices. In Section 3 we explain how these products can be priced using simulation techniques. In Section 4 we provide an application of our framework to a simple model and provide prices for the various products. Finally, Section 5 concludes.

## 2. Examples of longevity derivative products

In the financial markets several products exist that allow investors to transfer risk. A classic example is the use of forwards or futures contracts on stock indices to hedge exposure to the equity markets or to speculate in the future direction of the market. Futures contracts also exist on bonds and other interest rate products. Swap contracts are also extremely important in the interest rate and foreign currency markets. A simple fixed for floating interest rate swap, for example, is a contract in which one counterparty makes fixed interest rate payments and in return receives floating rate payments on a given notional amount.

Contrary to the aforementioned products that not only exist but are also liquidly traded in many markets, derivative products that allow us to hedge or transfer longevity risk virtually do not exist or, if they do exist conceptually, they do not trade actively. In this section we review some financial products that could be introduced in the market for longevity risk. The first of these is the survivor forward, which is arguably the simplest possible type of financial derivative product. It can be used to form the basic building blocks upon which many more complex derivatives can be constructed. One example is the survivor swaps which we detail next. Finally, we discuss how option contracts can be constructed where the underlying asset is longevity risk.

2 Commonly used by insurance companies, the Lee–Carter model (see Lee and Carter, 1992) is a simple one-factor model that offers a relatively good fit over a full range of ages (see Gaille, 2010, for more details). Li and Chan (2007) even call the Lee–Carter model the "gold standard of mortality trend fitting and forecasting". This one-factor model estimates and forecasts mortality rates on the assumption that mortality is perfectly correlated at all ages and prevents any cohort-effect, an effect that is specific to a particular year of birth.