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Model and heuristic algorithm of the joint replenishment problem with complete backordering and correlated demand

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ABSTRACT

In practice, demands for items are often correlated with each other because of cross-selling effects. Traditional inventory models hold the assumption that demands are independent, which may lead to the risk of misestimating the inventory cost of associated items. This paper addresses a multi-item inventory system where demands for some minor items are correlated with that for a major item. Because of the cross-selling, demand for minor items may either be raised by the sales of the major item or be pulled down by the unsatisfied demands for the major item. Thus, a joint inventory policy considering correlations across items should be pursued. Considering general integer policy, we propose a joint replenishment problem (JRP) model with complete backordering and correlated demand. Through the use of derivatives the model is transformed to minimize a cost function with respect to multiples of the major item's order cycle. Given that the optimal solution should offer a trade-off between the ordering cost and the inventory holding cost of each item, we develop a heuristic algorithm by adjusting the replenishment frequencies of minor items to solve the model. The heuristic is tested by simulated numerical examples and shows satisfactory results.

1. Introduction

Stochastic demand is an uncertain factor that has been considered in many risk-related models of inventory management. For instance, Luciano et al. (2003) employed value-at-risk (VaR) as a risk measure for inventory management in a static multiperiod framework. Tapiero (2005) studied a specific problem based on risk exposure in inventory management considering shortage cost. Jammernegg and Kischka (2009) proposed a newsvendor model for risk-averse and risk-taking inventory managers pursuing a robust inventory decision.

On the other hand, it has been generally recognized that cross-selling effects will lead to demands for items being correlated with each other, which can be depicted by the probability with which an item can be demanded together with an associated item. In fact, cross-selling implies the very frequent phenomenon in retail shops or supermarkets whereby some items are always purchased jointly with a given probability by customers due to their unknown interior associations (Anand et al., 1997; Kleinberg et al., 1998). Because of cross-selling effects, demand for an item can either be raised by the gained sales of its associated item or be pulled down by shortages in the associated item (Brijs et al., 1999; Wong et al., 2005). The cross-selling effect can be regarded as another uncertain factor that can also lead to the risk in inventory management when demand cannot be satisfied directly from shelf stock.

In a single-item inventory system, when stockouts occur, only an independent item is considered. In this case, a frequent context is that unmet demand during the stockout period can be totally backordered and satisfied in the next replenishment. If all the unmet demands can be backordered, the manager will be confronted with the economic order quantity (EOQ) problem with complete backorders, which has been extensively studied (Grubbström and Erdem, 1999; Cárdenas-Barrón, 2001; Ronald et al., 2004; Sphicas, 2006; Minner, 2007). Recently, Cárdenas-Barrón (2009) proposed a single-stage economic production quantity (EPQ) model with planned backorders considering the presence of defective products. The author assumes that all backorders are completely satisfied to determine the optimal production batch size and backordered quantity for minimizing the total cost per unit time. Cárdenas-Barrón (2010) further presented a method for determining the optimal lot size and backorders utilizing the two well-known inequalities: the arithmetic–geometric mean inequality (AGM) and the Cauchy–Bunyakovsky–Schwarz (CBS) inequality. The advantage of their method is to optimize the inventory model without derivatives.

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The topic of the single-item inventory model with backorders has attracted considerable attention in inventory management. However, it is more familiar for multiple items to occur in real-world inventory systems, which makes it inevitable for inventory managers to be confronted with multi-item problems in inventory management. Moreover, demands for items are frequently correlated with each other. Thus, we should take into account multiple items in inventory management by introducing correlated demand into inventory models.

The basic formulation of the multi-item inventory model is the so-called joint replenishment problem (JRP) (Goyal, 1973; Silver et al., 1998). In the JRP, it is assumed that many items are procured from the same producer. Any replenishment can give rise to a major ordering cost, and at the same time a minor ordering cost is always associated with the item replenished. For instance, if we order different types of fish, the transportation will incur a major ordering cost because transportation costs always occur at every replenishment point regardless of what kind of fish we ordered. However, the keeping costs of different types of fish during transportation are only correlated to the specific type of fish and are charged only when the corresponding type of fish is replenished. Hence, in the JRP, many items can share a common major ordering cost if replenished simultaneously, and an item also incurs a minor ordering cost if ordered. Obviously, coordination of replenishments of all items reduces the number of major and minor ordering costs, which will lead to a lower total inventory cost. Furthermore, because supply cycles of all items must be coordinated for convenient communication and scheduling, it is a conventional assumption in the JRP that the time interval between replenishments of an item is a positive integer multiple of a basic order cycle.

The traditional JRP has been studied extensively including as heuristic algorithms and global optimal algorithms (Goyal 1973, 1974, 1985, 1988; Silver, 1976; Goyal and Belton, 1979; Kaspi and Rosenblatt, 1983, 1991; Goyal and Deshmukh, 1993; Viswanathan, 1996; Wildeman et al., 1997; Lee and Yao, 2003; Nilsson et al., 2007; Porras and Dekker, 2008), where independent demand and no stockouts are assumed. However, some inventory systems may adopt a backordering policy, which violates the assumption of no stockouts. Moreover, correlated demands across items caused by cross-selling frequently occur in practice (Zhang et al., 2011). Therefore, it is more applicable to build a multi-item inventory model taking into account of correlations across items.

This paper studies a multi-item inventory system comprising a major item and some minor items. Demand for the major item is independent and demand for the minor items is correlated with that for the major item. Because of cross-selling effects, if the major item is stocked out, customers will probabilistically give up buying the minor items and demand for the minor items may be pulled down to a certain degree. We assume that the unmet demand for the major item can be completely backordered, while demand for the minor items must be met without stockouts. Based on the general integer (GI) policy whereby the minor item's supply cycle is equal to a positive integer times the time length of the major item's supply cycle, we build a JRP model with complete backordering and correlated demand. Similar to the traditional JRP, the analytical solution of the proposed model cannot be found. Thus, we propose a heuristic algorithm that balances the ordering cost and the inventory holding cost by adjusting the replenishment frequencies of minor items to optimize the total inventory cost. The computational study shows that the solution attained by the heuristic is very close to the exact optimal solution.

The rest of the paper is organized as follows. In Section 2, we present our assumptions and notations and then build the JRP model with complete backordering and correlated demand caused by cross-selling. The heuristic algorithm for solving the model is developed in Section 3. Section 4 presents numerical examples for computational experiments and for investigating the effectiveness of the heuristic. Section 5 concludes the paper.

2. Model formulation

2.1. Assumptions and notations

The phenomenon whereby demand for some minor items is correlated with that for a major item often occurs in electronic supermarkets. For instance, it is highly likely that a customer who buys a digital camera will also buy an additional memory stick and a battery/cell simultaneously as standbys. Thus, sales of digital cameras often influence demand for memory sticks and batteries/cells. On the other hand, if the digital camera is stocked out, the customer will not purchase the memory stick and the battery as standbys until the digital camera is replenished. This phenomenon whereby sales of a major item may lead to demand for minor items is known as cross-selling. Because of cross-selling effects, the minor item can be either sold independently or promoted through joint sales with the major item.

Generally, if the major item is out of stock, because of monopolization, almost all customers will be willing to backorder the item and wait for the next replenishment. Thus, the inventory decision for the major item can be made according to the EOQ with complete backordering. However, if a minor item is stocked out, most customers will opt to purchase it from another vendor because the minor item may be cheaper or less technologically complex than the major item, which makes it difficult to monopolize the minor item. Therefore, the backordering rate of the minor item is nearly 0, which is a case of pure lost sales. From the single-item EOQ with pure lost sales, to maximize the profit on a minor item that is profitable but with a backordering rate equal to 0, we should meet all the demand without stockouts (Pentico and Drake, 2009). Since demands for the major and the minor items are correlated during the entire supply cycle, a joint inventory policy should be pursued for these items to maximize the profit from inventory management. In modeling this inventory system, the following assumptions are made.

1. The demand rate is constant and the planning horizon is infinite.
2. Replenishment is instantaneous and costs no time.
3. Lead time is zero.
4. The major item can be completely backordered and the minor items should not be stocked out.
5. Although it is probable that a customer will give up purchasing minor items while the major item is stocked out, we depict this in an expectation, i.e., we assume that one unit of lost sale of the major item will pull down the demand for the minor item at a constant rate.
6. For regular communication and easier scheduling, a general integer policy is employed, i.e., the time length of the order cycle of the minor item is equal to a positive integer times the time length of the order cycle of the major item.

The last assumption is based on the fact that the purchasing cost of the major item is often higher than that of the minor item. In practice, the expensive items, e.g., items in the A class, are often important in inventory and may be replenished more frequently to keep a lower inventory holding cost (Silver et al., 1998). Therefore, it is reasonable to assume that the time length of the supply cycle of a minor item is longer than that of a major item. Suppose that we have one major item and N minor items then, all the notations in the model are as follows.