Capital adequacy tests and limited liability of financial institutions

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1. Introduction

The theory of acceptance sets and risk measures occupies an important place in current debates about solvency regimes in both the insurance and the banking world. The objective of this paper is to investigate the class of surplus-invariant acceptance sets and their associated risk measures. These acceptance sets are characterized by the fact that acceptability does not depend on the positive part, or surplus, of a capital position. Further down we argue that surplus-invariant acceptance sets and risk measures have properties that are natural from the perspective of external risk measures, a term coined in Kou et al. (2013) to denote risk measures used for external regulation rather than for purely internal risk management purposes. In fact, risk measures with related properties have been recently introduced and studied independently by Cont, Deguest & He in Cont et al. (2013), who called them loss-based and in Staum (2013), whose author uses the term excess invariant.

1.0.1. Surplus-invariant acceptance sets

Throughout this paper capital positions – assets net of liabilities – of financial institutions are represented by the elements of \( L^+ \), the space of essentially bounded random variables on a probability space \((\Omega, \mathcal{F}, P)\). An institution is said to be adequately capitalized if its capital position belongs to a pre-specified acceptance set, i.e. to a nonempty, strict subset \( \mathcal{A} \) of \( L^+ \) such that \( Y \in \mathcal{A} \) whenever \( Y \approx X \) almost surely for some \( X \in \mathcal{A} \).

For a financial institution with limited liability, the positive and negative parts of a capital position \( X \) have a clear financial interpretation. The positive part \( X^+ := \max(X, 0) \) is called the surplus and represents the funds available to the owners after meeting liabilities. The negative part \( X^- := \max(-X, 0) \) represents the owners’ option to default, i.e. the amount by which the institution falls short of meeting liabilities. As explained in more detail in Section 2, when assessing the capital adequacy of a financial institution, regulators – acting on behalf of liability holders – should be indifferent to the amount of surplus, since it exclusively affects the owners and not the liability holders. This is the rationale for investigating surplus-invariant acceptance sets, i.e. acceptance sets \( \mathcal{A} \subset L^+ \) that have the following property:

\[
X \in \mathcal{A}, \ Y^+ = X^+ \Rightarrow Y \in \mathcal{A}. \tag{1.1}
\]
The main focus of the paper is on surplus-invariant acceptance sets that are convex or coherent. These acceptance sets are important because they reflect the principle that higher diversification should lead to lower capital requirements. In Theorem 4.1 we provide a dual characterization of convex, surplus-invariant acceptance sets under a standard closedness assumption. As an application, we show in Theorem 4.5 that the only coherent acceptance sets that are surplus invariant are essentially scenario-based acceptance sets, i.e. acceptance sets of the form

\[ \text{SPAN}(A) := \{ X \in L^\infty; X1_A \geq 0 \text{ almost surely} \}, \]  

(1.2)

for some scenario set \( A \in \mathcal{F} \). Therefore, if we simultaneously insist on coherence and surplus invariance, we end up with an extremely limited choice of acceptability criteria that have properties which may not always be desirable. Indeed, if \( A \neq \Omega \), then \( \text{SPAN}(A) \) is blind to the behaviour of a capital position outside of \( A \) and, if \( A = \Omega \), we end up requiring a zero default probability for an institution to be acceptable. Hence, if we view surplus invariance as a desirable property, to obtain a wider range of acceptability criteria we need to abandon coherence. The situation is even more extreme if law invariance is additionally required, as shown in Corollary 4.6: The only coherent acceptance set that is simultaneously law invariant is the positive cone \( L^+ \). In particular, acceptence sets based on Expected Shortfall are not surplus invariant. This discussion has implications for the other standard acceptability tests used in practice, are surplus invariant, though not convex. It is worthwhile noting that acceptance sets based on Value-at-Risk, Shortfall risk measure that is cash additive subject to positivity can be expressed as a truncated risk measure of the form \( \max(\rho_{AS}, 0) \) where \( A \) is surplus invariant and \( S = (S_0, S_T) \) is a traded asset with initial price \( S_0 > 0 \) and positive payoff \( S_T \in L^+ \). The quantity \( \rho_{AS}(X) \) has an explicit operational meaning. Indeed, when finite and positive, it represents the amount of capital that can be extracted from the institution, by shorting \( S \) and dividend out the proceeds, without compromising its capital adequacy.

When \( A \) is surplus invariant, we show in Proposition 3.10 that the risk measure \( \rho_{AS} \) enjoys the following property:

\[ \rho_{AS}(X) = \rho_{AS}(-X') \text{ for all } X \in L^\infty \text{ with } \rho_{AS}(X) \geq 0. \]  

(1.6)

Note that the above equality requires that \( \rho_{AS}(X) \geq 0 \). Thus, for unacceptable positions with the same negative part capital requirements are identical. By contrast, for acceptable positions, the amount of capital that can be extracted without compromising acceptability will typically not depend only on the negative part, but also on how far into acceptability the position is. In Proposition 4.4 we investigate the impact of surplus invariance on the dual representation of risk measures defined by (1.5).

In the last section of the paper we clarify the link between risk measures of the form \( \rho_{AS} \) and the loss-based risk measures studied in Cont et al. (2013) as well as the shortfall risk measures considered in Staum (2013). We show that, with the exception of scenario-based risk measures, the acceptance set induced by all explicit examples of loss-based risk measures considered in Cont et al. (2013) collapses down to the positive cone \( L^+ \). This is a consequence of Proposition 5.14. As already noted, from a capital adequacy perspective this is quite severe since, in this case, the only adequately capitalized institutions would be those that can never default.

An obvious way to vest a positive risk measure \( \rho \) with an operational meaning, which is briefly mentioned in Cont et al. (2013), is to require that raising and holding the capital amount \( \rho(X) \) in cash be sufficient to ensure acceptability. This leads to the concept of a cash compatible risk measure, i.e. a risk measure \( \rho \) satisfying:

\[ \rho(X + \rho(X)) = 0 \text{ for all } X \in L^\infty. \]  

(1.7)

Under the assumption of cash compatibility, we prove in Theorem 5.22 that every convex loss-based risk measure and every shortfall risk measure that is cash additive subject to positivity can be expressed as a truncated risk measure of the form \( \max(\rho_{AS}, 0) \).

The paper is structured as follows. In Section 2 we present background material on acceptance sets and risk measures. In Section 3 we introduce the concept of surplus invariance, discuss the main examples and provide a simple characterization of this property. In Section 4 we focus on dual representations and the resulting characterization of coherent, surplus-invariant acceptance sets, which is the main result of this paper. Finally, in Section 5 we discuss the link to the risk measures considered in Cont et al. (2013) and in Staum (2013). All proofs have been collected in a final appendix.
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