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A tabu search meta-heuristic approach to the dual response systems problem

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ABSTRACT

This paper presents an alternative approach to the dual response systems problem by utilizing a tabu search algorithm that yields a string of solutions and examine the trade-offs graphically and systematically how the controllable variables simultaneously impact the mean and the standard deviation of a characteristic of interest relevant to an industrial process. Heuristic-based search techniques may be very useful for cases where interactive multi-objective optimization techniques are not available due to lack of willingness of decision-makers. A further advantage of tabu search is its simplicity and we show that the entire process only occupies a few lines of codes and generates string of solutions in speedy manner especially for the larger-the-better/smaller-the-better cases of Taguchi's robust parameter design. The procedure is illustrated with an example.

1. Introduction

In today's increasingly competitive marketplace more attention is being paid to off-line quality control and the idea of robust product design. Recent advances in quality technology have resulted from considering the variation of a quality characteristic as well as its mean value. Taguchi and Wu (1985) and Taguchi (1986) have been a major proponent of this philosophy. The recent push for quality improvement in industry has brought response surface methodology (RSM) to the attention of many users (Khuri, 1996). In the 1990s, much attention was given to the optimization of dual response systems (DRS) as an important RSM tool for quality improvement. In our context, the dual response refers to the mean and the standard deviation of the process.

Taguchi's robust parameter design (RPD) calls for simultaneous optimization of the mean and standard deviation responses. The RPD problem is a special case of the multiple response problems, where two responses, the mean and variance of a fundamental response/characteristic observed during the experiment. The DRS problem requires an overall optimization – that is a simultaneous satisfaction with respect to both the mean and the standard deviation of a quality characteristic. Basically one builds two empirical response surface models – one for the mean and one for the standard deviation – and then optimizes one of the responses subject to an appropriate constraint on the other's value. The decision how to use the dual response approach to achieve the basic goals of Taguchi's philosophy depends upon the ultimate purpose of the experiment. Vining and Myers (1990) adapted to the three basic cases of RPD, i.e., "larger-the-better", "smaller-the-better", and "target-is-best". In the larger-the-better/smaller-the-better cases, one seeks the settings of the control parameters that maximize/minimize the mean response while controlling the standard deviation at some specified value. In the target-is-best case, one is interested in minimizing the standard deviation while keeping the mean response at a specified target value. In each of three cases, a solution is found under an additional constraint on the vector of control variables. Let $\mathbf{x} = [x_1, \ldots, x_k]$ be a $k \times 1$ vector of control variables. If a factorial type design is used for the purpose of experimentation, then a cuboidal region, defined by $-1 \leqslant x_i \leqslant 1, i = 1, 2, \ldots, k$ (k is the number of control variables), may be a good choice for defining the region of interest. When using a spherical type design (e.g., a central composite design), the additional constraint is defined by $\mathbf{x}'\mathbf{x} \leqslant \rho^2$ where ρ is the design radius.

A major drawback of selecting the most dominant response as the objective function and then taking the other one as a constraint (i.e., single objective optimization) imposes an unnecessary restriction on the value of secondary response especially when dealing with the larger-the-better/smaller-the-better cases. Keeping the standard deviation below a specified value may rule out better conditions during the optimization process, since an acceptable value for the standard deviation response is usually unknown. In fact, process conditions that result in a smaller standard deviation are often preferable. Recently, Köksoy and Doganaksoy (2003) realized that the standard deviation of any performance property could be treated as a new property in its own right as far as Pareto optimizer was concern (i.e., multi-objective optimization). The interaction among different conflicting objectives gives rise to a string of

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solutions, called Pareto optimal solutions. Pareto solutions are those for which improvement in one objective can only occur with the worsening of at least one other objective. Thus, instead of a unique solution to the problem, the solution to a multi-objective problem is a (possibly infinite) set of Pareto points. Since none of these alternative solutions can be identified as better than others without any further examination, the goal in multi-objective optimization is to find as many alternative solutions as possible in speedy manner. Once such set of compromised solutions is found, it usually requires a higher level decision making with other considerations to choose one of them for implementation. We believe that such analysis is useful compared to a single optimal solution, and that is required in order to achieve an improved understanding of the problem before searching for a final optimal solution.

Even though we support and follow the main philosophy proposed by Köksoy and Doganaksoy (2003), their optimization method, namely the NIMBUS (Nondifferentiable Interactive Multi-objective Bundle-based Optimization System) algorithm, based on interactive articulation of preference information has the following difficulty in applications: the interest devoted to interactive methods can be explained by the fact that assuming the decision maker has enough time and capabilities for co-operation. The convergence is not necessarily fast if the decision maker is not purposeful. The freedom of the decision maker has both positive and negative aspects. The decision maker can direct the solution process and is free to change her or his mind during the process. Because of the subjectivity of the decision makers, different starting points, different types of questions or interaction styles may lead to different final solutions.

In order to overcome the difficulties associated with the optimization method of Köksoy and Doganaksoy, we propose a tabu search algorithm for finding the Pareto solutions for the DRS problem. First we convert the problem into a scalar one by using a weighted linear sum of the objectives and then optimize the weighted objective function. The major advantage of the proposed formulation is that it does not require any constraints on the secondary response. Unlike the NIMBUS method, the proposed tabu approach does not set any specific assumptions on the behavior or the preference structure of the decision maker. It means that the proposed method will still work and generate many alternative solutions whether or not the decision maker has enough time and capabilities for co-operation.

The rest of the paper is organized as follows. In the next section we present a revised problem formulation of the DRS problem. We then briefly review the fundamental concepts of tabu search. This is followed by a numerical example that illustrates the proposed approach. We conclude the paper with a summary. The proposed tabu search algorithm is described in the Appendix.

2. Revised problem formulation of the DRS problem

Assume that an appropriate 2nd order response surface experiment is conducted. Let $\hat{\mu}$ and $\hat{\sigma}$ denote the fitted response surfaces of the process mean and standard deviation, respectively. Assume that these responses may be modelled by

$$\hat{\mu} = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum \sum_{i<j}^{k} b_{ij} x_i x_j \tag{1}$$

$$\hat{\sigma} = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} a_{ii} x_i^2 + \sum \sum_{i<j}^{k} a_{ij} x_i x_j \tag{2}$$

where the b's and a's represent the estimated coefficients, and the x's are the control variables ($\mathbf{x} \in \mathbf{R}$, where $\mathbf{R}$ is a region of interest). Therefore, the DRS optimization problem for the *smaller-the-better* case may be defined as:

$$\text{Minimize}(w_1 \hat{\mu} + w_2 \hat{\sigma}) \tag{3}$$

$$\mathbf{x} \in \mathbf{R}$$

where w_1 and w_2 are pre-specified positive constants which are chosen based on the relative importance of the mean and standard deviation responses, usually obtained through the advice of experts on the process of interest. The objective is to find the settings of x's that would optimize the weighted objective function subject only to the constraint that defines the region of interest $\mathbf{R}$. As mentioned earlier, we consider two different regions of interest, cuboidal and spherical. For the *larger the better* case, the mean should be maximized therefore w_1 is replaced with $(-w_1)$ for solving the DRS problem using the tabu algorithm. The proposed formulation of the DRS problem is directly applicable for the smaller-the-better and the larger-the-better cases of RPD. The target-is-best case, however, needs a preset constraint on the value of the mean response function and, thus, the weighted objective optimization is not necessary. In somewhat limited manner by changing the weight values in Eq. (3) one can still find string of solutions for the target-is-best case, however, this case can be more directly addressed by a single objective optimization (e.g., Del Castillo and Montgomery (1993) or Copeland and Nelson (1996)).

3. A brief overview of tabu search

Tabu search is a local search-based metaheuristic method that has been successfully applied to a wide class of hard optimization problems. Appropriate subject areas include bioengineering, finance, manufacturing, scheduling, and political districting. It was first presented by Glover (1986) and also sketched by Hansen in 1986.

Tabu search uses a short-term memory structure called a 'tabu list'. A potential solution is marked as "tabu" so that the algorithm does not visit that possibility repeatedly. Tabu search starts with an initial solution. Algorithms based on tabu search perform a neighborhood search (i.e., a local search) starting from a current solution to its best neighbor (the one with the best objective value among all examined candidates). Tabu search modifies the neighborhood structure of each solution as the search progresses. All the neighbors of a current solution are examined and the best non forbidden move is selected. Note that this move may decrease the quality of the solution, but necessary in order to increase the likelihood of escaping from so-called local optimum "traps". A tabu list stores all the previously exploited moves or solutions which are now forbidden. The search continues until some stopping criterion has been satisfied. To avoid cycling during the search process, the reverses of the last certain number of moves, formed as a tabu list, are prohibited or announced as tabu restricted for certain number of iterations (i.e., the tabu duration). To prevent a too rigorous parameter settings of the tabu restriction, some aspiration criteria are usually introduced which allow overriding the tabu restriction and thereby to guide the search toward a promising region. Intensification and diversification strategies with tabu search are also applied to emphasize and broaden the search in the solution space, respectively. More discussion can be found in articles by Chao (2002), Vilcot and Billaut (2008), and Caserta and Uribe (2009).

The basic components of the tabu algorithm are outlined below:

1. *Configuration*: Coding of a solution.
2. *Move*: Selected feasible direction of the search.
3. *Set of candidate moves*: Feasible directions of the search.
4. *Tabu restrictions*: The length of the tabu list.
5. *Aspiration criteria*: Overriding the tabu restriction.
6. *Stopping condition*: Terminating the search.