Computers & Industrial Engineering 65 (2013) 286–296

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A hybrid meta-heuristic for multi-objective vehicle routing problems with time windows ☆

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ARTICLE INFO

Article history:
Received 29 December 2011
Received in revised form 1 December 2012
Accepted 15 January 2013
Available online 14 February 2013

Keywords:
Vehicle routing problems
Time windows
Travelling distance
Load imbalance
Multi-objective optimization
Hybrid meta-heuristics

ABSTRACT

The Capacitated Vehicle Routing Problem with Time Windows is an important combinatorial optimization problem consisting in the determination of the set of routes of minimum distance to deliver goods, using a fleet of identical vehicles with restricted capacity, so that vehicles must visit customers within a time frame. A large number of algorithms have been proposed to solve single-objective formulations of this problem, including meta-heuristic approaches, which provide high quality solutions in reasonable runtimes. Nevertheless, in recent years some authors have analyzed multi-objective variants that consider additional objectives to the distance travelled. This paper considers not only the minimum distance required to deliver goods, but also the workload imbalance in terms of the distances travelled by the used vehicles and their loads. Thus, MMOEASA, a Pareto-based hybrid algorithm that combines evolutionary computation and simulated annealing, is here proposed and analyzed for solving these multi-objective formulations of the VRPTW. The results obtained when solving a subset of Solomon's benchmark problems show the good performance of this hybrid approach.

1. Introduction

The Vehicle Routing Problem (VRP), and its multiple variants, is a core problem in transportation, logistics, and supply chain management. Logistics, and especially the distribution of goods, lies at the heart of business activity because it is often coupled with inventory and production decisions, and the delivery cost accounts for a significant portion of the total logistic costs (Alabas-Uslu & Dengiz, 2011). Bearing in mind that problems in the domain of goods distribution can be viewed as a VRP (Mester & Bräysy, 2007), this problem contributes directly to reducing costs of all logistic systems (Alvarenga, Mateus, & de Tomic, 2007). Logistic managers need to make decisions to improve the design of their logistic systems, including appropriate decisions concerning the strategies to provide customers with their services while satisfying the company's logistic priorities according to the available vehicle fleet. Furthermore, current concerns over global warming, resource depletion, and the social impact of traffic congestion and pollution are driving companies, governments, and researchers to improve the efficiency of logistics and distribution operations (Hosny & Mumford, 2010).

VRPs are combinatorial optimization problems linked with many branches of mathematics, economics, computer science, and operations research. Since the family of vehicle routing problems is included in the category of NP-hard problems (Lenstra & Rinnooy Kan, 1981), they are hard to solve, especially when the number of customers is large (Lee, Lee, & Lin, 2008). The richness and difficulty of these problems has made vehicle routing an area of intense research work, as witnessed by the large number of research papers found in the literature (Eksioglu, Vural, & Reisman, 2009). Often, the number of customers combined with the complexity of real-life data does not permit them to be solved using exact methods, which is why current research concentrates on heuristic algorithms that are capable of finding high quality solutions to real-life problems in limited time. In particular, heuristics and meta-heuristics support managers in decision-making with approximate solutions to complex problems (Gendreau & Potvin, 2010).

There are different variants of VRPs that aim to take into account the constraints and details of the problem, while also including different aspects of its nature, such as its dynamicity, time dependency, stochastic aspects, etc. An important variant of the VRP is the Capacitated Vehicle Routing Problem with (hard) Time

☆ This manuscript was processed by Area Editor Iris F.A. Vis.

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http://dx.doi.org/10.1016/j.cie.2013.01.007

Windows (VRPTW). It consists in determining the optimal set of routes of a fleet of identical vehicles with restricted capacity so that all customers, whose demands are known, are serviced exactly once within each time window. These time windows impose that the vehicle must begin the service to the customer within the time window defined by the earliest and latest times allowed by the customer for the start of service (El-Sherbeny, 2010). Routing problems are often set up with the single-objective of minimizing the cost of the solution despite the fact that the majority of the real applications associated with this problem are multi-objective in nature (Jozefowiez, Semet, & Talbi, 2008; Dabia, Talbi, van Woensel, & de Kok, 2013).

This paper presents a new Pareto-based multi-objective approach that uses a multi-start simulated annealing strategy for solving a multi-objective formulation of the VRPTW that aims to minimize the total distance of the vehicles used to service the customers, while also minimizing the imbalance of workloads (distances travelled/goods delivered by the vehicles). This new approach is evaluated in comparison with two well-known multi-objective evolutionary algorithms, NSGA-II (Deb, Agrawal, Pratap, & Meyarivan, 2001) and SPEA2 (Zitzler, Laumanns, & Thiele, 2001). Section 2 describes the Capacitated Vehicle Routing Problem with Time Windows, while also justifying the importance of using multi-objective optimization methods to solve real-life routing problems. Section 3 details the multi-objective evolutionary algorithm proposed here, while the results obtained when solving some test problems are commented upon in Section 4. Finally, Section 5 provides the conclusions to this work.

2. The Capacitated Vehicle Routing Problem with Time Windows

Since Dantzig and Ramser (1959) introduced the VRP, it has been one of the most widely analyzed NP-hard problems. The basic VRP has been extended to include aspects such as characteristics of the network, the fleet, and the customers, making the problem more difficult to be solved (Bochtis & Sørensen, 2009). The typical formulation of the single-objective VRPTW involves the routing of a set of vehicles with identical capacity stationed at a central depot (logistic centre) which operate within certain time windows and are used to visit and fully supply the demands of a set of customers. Routes are designed to start and end at the depot and the total demand met by any route cannot exceed the vehicle capacity. The customers, whose demand can only be supplied once by exactly one vehicle, are located in diverse geographical regions and have predefined requirements of goods and a service time. The depot on the one hand, and each customer on the other, have time windows, implying that the vehicle may arrive before the time window opens but not after it has closed, and the customer cannot be serviced until the time windows open. The distances between customers are measured by Euclidean distances, and the total distance travelled by all the vehicles defines the travelling times. Therefore, the single-objective VRPTW aims to determine which customers are visited by each vehicle and the route each vehicle follows to serve the assigned customers, while the distances travelled by the vehicles are minimized and the capacity and time windows constraints are satisfied. The VRPTW has been widely studied because it remains one of the most difficult problems in combinatorial optimization and has a considerable economical impact on all logistic systems (Alvarenga et al., 2007), especially due to the importance of just-in-time production systems and the increasingly tight coordination of supply chain operations (Figliozzi, 2010).

Some exact methods have been proposed for the VRPTW, including Lagrangian relaxation-based methods, column generation, and dynamic programming (El-Sherbeny, 2010). However, exact methods often perform poorly in intermediate and large problem instances, especially in some VRP variants (Kritikos & Ioannou, 2010). As a result, several heuristic and meta-heuristic methods have been proposed for solving the VRPTW, including multi-start local search (Bräysy, Hasle, & Dullaert, 2004), genetic algorithms (Alvarenga et al., 2007), tabu search (Cordeau, Laporte, & Mercier, 2001), etc., and the results obtained show that these methods obtain acceptable results in reduced runtimes.

2.1. Mathematical model for VRPTW

The VRPTW can be modeled on an non-directed complete graph $G(V,E)$ where vertices $V=\{1,\ldots,N\}$ correspond to the depot and the customers, and edges $e \in E\{(i,j){:}\ i,j \in V\}$ to the links between them (El-Sherbeny, 2010).

Decision variable

$$X_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

Parameters

a_j	is the earliest time for customer j to allow the service,
b_j	is the latest time for customer j to allow the service,
C_{ij}	is the cost for travelling from node i to node j (here, C_{ij} is considered as the distance or time required for travelling from node i to node j),
d_j	is the demand at customer j,
K	is the maximum number of vehicles that can be used,
N	is the number of customers plus the depot (the depot is denoted with number 1, and the customers are denoted as $2,\ldots N$),
Q	is the loading capacity of each vehicle.

The VRPTW can be stated as follows:

$$\text{minimize}: \quad TD = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^k C_{ij} \tag{1}$$

$$\text{subject to} \quad X_{ii}^k = 0 \quad (\forall i \in \{1,\ldots,N\}, \quad \forall k \in \{1,\ldots,K\}) \tag{2}$$

$$X_{ij}^k \in \{0,1\} \quad (\forall i,j \in \{1,\ldots,N\}, \quad \forall k \in \{1,\ldots,K\}) \tag{3}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{N} X_{ij}^k = 1 \quad (\forall j \in \{2,\ldots,N\}) \tag{4}$$

$$\sum_{i=1}^{N}\sum_{j=2}^{N} X_{ij}^k d_j \leqslant Q \quad (\forall k \in \{1,\ldots,K\}) \tag{5}$$

$$\sum_{k=1}^{K}\sum_{j=2}^{N} X_{1j}^k \leqslant K \tag{6}$$

$$\sum_{j=2}^{N} X_{1j}^k - \sum_{j=2}^{N} X_{j1}^k = 0 \quad (\forall k \in \{1,\ldots,K\}) \tag{7}$$

$$a_j \leqslant s_{kj} \leqslant b_j \quad (\forall i,j \in [1,\ldots,N], \quad \forall k \in \{1,\ldots,K\}) \tag{8}$$

$$s_{ki} + C_{ij} - L\left(1 - X_{ij}^k\right) \leqslant s_{kj} \quad (\forall i,j \in \{1,\ldots,N\}, \quad \forall k \in \{1,\ldots,K\}) \tag{9}$$

Eq. (1) is the objective function of the problem. Eq. (2) denotes that a vehicle must travel from one node to a different one. Eq. (3) indicates that X_{ij}^k is equal to 1 if vehicle k goes from node i to node j, and is equal to 0 otherwise. Eq. (4) states that a customer is visited once by exactly one vehicle. By specifying the constraint of Eq. (5), it is taken into account that for a given vehicle k, the load that has to be transported to complete the route assigned to such vehicle