Finite Elements in Analysis and Design 64 (2013) 1–12

Contents lists available at SciVerse ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Integrated optimization of structural topology and control for piezoelectric smart plate based on genetic algorithm

B. Xu [a,*], J.P. Ou [b], J.S. Jiang [a]

[a] School of Mechanics, Civil & Architecture, Northwestern Polytechnical University, Xi'an, 710072, PR China
[b] Dalian University of Technology, Dalian, 518055, PR China

ARTICLE INFO

Article history:
Received 14 October 2007
Received in revised form
25 May 2012
Accepted 20 September 2012
Available online 22 October 2012

Keywords:
Integrated optimization
Structural topology
Vibration control
Genetic algorithm
Piezoelectric smart plate

ABSTRACT

The integrated optimization of structural topology, number and positions of the actuators and control parameters of piezoelectric smart plates is investigated in this paper. Based on the optimal control effect in the independent mode control and singular value decomposition of the distributed matrix of total performance index for all physical control forces for piezoelectric smart plate, a new criterion, where several large values in singular values are selected, is put forward to determine the optimal number of the assigned actuators in the coupled modal space control. Furthermore, the optimal positions of actuators are ascertained by singular value decomposition of the modal distributing matrix. The integrated optimization model, including the optimized objective function, design variables and constraint functions, is built. The design variables include the logic design variables of structural topology, the number and positions of actuators as well as the control design parameters. Some optimal strategies based on genetic algorithm (GA), such as structural connection checking and structural checkerboard checking and repairing technique, are used to guide the optimization process efficiently. The results of two numerical examples show that the proposed approach can produce the optimal solution with clear structural topology and high control performance.

1. Introduction

In recent years, because more and more stringent performance requirements are imposed in advanced engineering application, considerable attention has been paid to structural vibration control. There are two classical routes to suppress/reduce structural vibration. One is to implement vibration control, such as passive control, semi-active control and active control. The other is to implement structural dynamic optimization, including size optimization, shape optimization and topology optimization. Especially, the dynamic topology optimization becomes the research emphasis up to now. Xu et al. [1] put forward topology group concept for truss topology optimization with frequency constraint, where nodal mass is taken into consideration. Jog [2] proposed the global measure and local measures for minimizing the vibrations of structures subjected to periodic loading. Rong et al. [3] presented the topology optimization of continuous structures under stochastic excitations based on ESO. Guan et al. [4] optimized cable-supported bridges with frequency constraint incorporating 'nibbling' technique. Du et al. [5] dealt with the topology optimization problems formulated directly with the design objective of minimizing the sound power validated from the structural surfaces into a surrounding acoustic medium.

Traditionally, structural dynamic optimization and vibration control are separately carried out so that the optimal control effect cannot obtained, i.e., structural parameters are optimized first and then an optimal controller is designed. How to deal with the strong coupling between structural dynamics and active control is a well-recognized challenge for the design of piezoelectric smart structures. At present, the problem has received much attention in the field of piezoelectric structural design. Some papers deal with the integrated optimization of structure and control in order to acquire hybrid optimization effect. Xu et al. [6,7] studied the integrated optimization of structure and control for piezoelectric intelligent trusses, in which actuators/sensors positions are also taken as the design variables. Zhu et al. [8] investigated simultaneous optimization with respect to the structural topology, actuator locations and control parameters of an actively controlled plate structure, where the topology design variables are relaxed to take all values between 0 and 1 and structural and control design variables are not treated within same framework. But most of these researches mainly focus on simple designs such as the integrated optimization of structural size and control, where the structural topology is predetermined. The integrated optimization problem of structural topology and control has not been extensively treated despite it is very important, including the number and positions of actuators.

* Corresponding author. Tel.: +86 29 8849 5992; fax: +86 29 8843 1021.
E-mail addresses: xubind@sina.com, xubin@nwpu.edu.cn (B. Xu).

0168-874X/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.finel.2012.09.004

This paper is organized as follows: in Section 2, based on the optimal control effect for the control design in the independent modal space, a new method based on singular value decomposition is presented determine the optimal number and positions of actuators. The integrated topology optimization model, including the design variables, the objective function and the constraint functions, is built in Section 3. Section 4 introduces the corresponding optimization algorithm combing genetic algorithm with the checking technique of structural topology effectivity. Finally in Section 5, two numerical examples are used to highlight and demonstrate the validity of the proposed method.

2. Control design

For control design, we first obtain the optimal modal control in the independent modal space. Further, in coupled modal space control, based on singular value decomposition of the distributing matrix of the total performance index for all physical control forces, the corresponding criterion is build to determine the optimal number of actuators. Then, the singular value decomposition of the modal distributing matrix is applied to ascertain the optimal positions of the assigned actuators, and the required input voltages or physical forces of the assigned actuators are recalculated.

The dynamic behavior of vibration control for the piezoelectric smart plate under initial disturbance is then determined by the equations

$$\begin{cases} M\ddot{\boldsymbol{x}}(t)+C\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{x}(0)=\boldsymbol{x_0} \quad \dot{\boldsymbol{x}}(0)=\dot{\boldsymbol{x}}_0 \end{cases} \tag{1}$$

where $\boldsymbol{x}(t)\in R^n$, $\dot{\boldsymbol{x}}(t)\in R^n$and $\ddot{\boldsymbol{x}}(t)\in R^n$ are system nodal displacement, velocity and acceleration vectors, respectively, and n is the number of the degrees of freedom of the controlled system; $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are system mass matrix, damping matrix and stiffness matrix, respectively; $\boldsymbol{B}$ is the actuator distributing matrix and the corresponding feasible domain is Ω: $\boldsymbol{B}_{n\times n_a}$, where the element of $\boldsymbol{B}_{n\times n_a}$ is 0 or 1; $\boldsymbol{u}(t)\in R^{n_a}$ is the control force vector and n_a is the number of assigned actuators; $\boldsymbol{x}_0$ and $\dot{\boldsymbol{x}}_0$ are the initial displacement and velocity disturbances, respectively.

2.1. Step 1: Independent modal space control

By coordinate transformation, Eq. (1) can be changed into the following modal form:

$$\ddot{y}_i(t)+2\xi_i\omega_i\dot{y}_i(t)+\omega_i^2 y_i(t)=\boldsymbol{\varphi}_i^T\boldsymbol{B}\boldsymbol{u}(t), \quad i=1,2,\ldots,n \tag{2}$$

where $y_i(t)$, $\dot{y}_i(t)$ and $\ddot{y}_i(t)$ are the modal displacement, the modal velocity and the modal acceleration of the ith mode, respectively; ω_i and $\boldsymbol{\varphi}_i$ are the ith natural frequency and the corresponding mode, respectively; ξ_i. is the ith damping ratio.

The state-space equations for the ith controlled mode corresponding to Eq. (2) can be expressed as

$$\dot{\boldsymbol{Z}}_i(t)=\boldsymbol{A}_i\boldsymbol{Z}_i(t)+\boldsymbol{B}_i f_i(t), \quad i=1,2,\ldots,n_m \tag{3}$$

where

$$\boldsymbol{Z}_i(t)=\begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \end{bmatrix}, \quad \boldsymbol{A}_i=\begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix},$$
$$\boldsymbol{B}_i=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad f_i(t)=\boldsymbol{\varphi}_i^T\boldsymbol{B}\boldsymbol{u}(t) \tag{4}$$

and n_m is the number of the controlled modes.

We introduce the quadratic objective function as

$$J_i=\frac{1}{2}\int_0^\infty \left(\boldsymbol{Z}_i^T(t)\boldsymbol{Q}_i\boldsymbol{Z}_i(t)+r_i f_i^2(t)\right)dt \tag{5}$$

where $\boldsymbol{Q}_i$ is a positive semi-definite matrix, and r_i is a positive weighting factor; Assuming $\boldsymbol{Q}_i=\begin{bmatrix} \omega_i^2 & 0 \\ 0 & 1 \end{bmatrix}$, J can be regarded as the sum of modal potential energy and modal kinetic energy corresponding to the ith mode as well as the performance index of input energy for the ith modal control force.

The minimization of Eq. (5) with respect to f_i results in

$$f_i(t)=-r_i^{-1}\boldsymbol{B}_i^T\boldsymbol{P}_i\boldsymbol{Z}_i(t) \tag{6}$$

where $P_i\in R^{2\times 2}$ is the positive semidefinite symmetric matrix, and satisfies the following Riccati equation

$$\boldsymbol{P}_i\boldsymbol{A}_i+\boldsymbol{A}_i^T\boldsymbol{P}_i+\boldsymbol{Q}_i-r_i^{-1}\boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{B}_i^T\boldsymbol{P}_i=\boldsymbol{0} \tag{7}$$

Substituting Eq. (6) into Eq. (3), the state equation for the closed-loop system can be expressed as

$$\dot{\boldsymbol{Z}}_i(t)=\overline{\boldsymbol{A}}_i\boldsymbol{Z}_i(t) \tag{8}$$

where $\overline{\boldsymbol{A}}_i=\boldsymbol{A}_i-r_i^{-1}\boldsymbol{B}_i\boldsymbol{B}_i^T\boldsymbol{P}_i$

The complex eigenvalues corresponding to Eq. (8) are expressed as

$$\lambda(\overline{\boldsymbol{A}}_i)=\overline{\sigma}_i\pm j\omega_{di} \tag{9}$$

where $\overline{\sigma}_i$ and ω_{di} are the real and imaginary parts of the ith complex eigenvalue, respectively, and $j=\sqrt{-1}$.

Based on the complex mode theory, the natural frequency and the damping ratio of the ith complex mode for the closed-loop system are defined as

$$f_{di}=\omega_{di}/2\pi \tag{10}$$

$$\xi_i=-\overline{\sigma}_i/\sqrt{\overline{\sigma}_i^2+\omega_{di}^2} \tag{11}$$

Obviously, the damp ratio is larger, the vibration is decayed more quickly and correspondingly the efficiency of suppressing vibration becomes better.

2.2. Step 2: Optimal number of actuators in coupled modal space control

Then, the total performance index of input energy for the required modal control forces corresponding to all controlled modes independent modal space control can be written as

$$J_f=\frac{1}{2}\sum_{i=1}^{n_m}\int_0^\infty r_i f_i^2(t)dt \tag{12}$$

In order to obtain the same control effect, the input energy for the required modal control forces corresponding to all controlled modes in coupled modal space must equal to that in independent modal space, i.e.,

$$\begin{aligned} J_f'=J_f&=\frac{1}{2}\sum_{i=1}^{n_m}\int_0^\infty r_i f_i^2(t)dt=\frac{1}{2}\int_0^\infty \boldsymbol{f}(t)\boldsymbol{R}\boldsymbol{f}^T(t)dt \\ &=\frac{1}{2}\int_0^\infty (\boldsymbol{\varphi}_c\boldsymbol{B}\boldsymbol{u}(t))\boldsymbol{R}(\boldsymbol{\varphi}_c\boldsymbol{B}\boldsymbol{u}(t))^T dt \\ &=\frac{1}{2}\int_0^\infty \boldsymbol{u}(t)(\boldsymbol{\varphi}_c\boldsymbol{B})\boldsymbol{R}(\boldsymbol{\varphi}_c\boldsymbol{B})^T\boldsymbol{u}^T(t)dt \\ &=\frac{1}{2}\int_0^\infty \boldsymbol{u}(t)\boldsymbol{R}'\boldsymbol{u}(t)^T dt \\ &=\frac{1}{2}\sum_i\sum_j\int_0^\infty \boldsymbol{R}_{ij}' u_i(t)u_j(t)dt \end{aligned} \tag{13}$$

where

$$\boldsymbol{f}(t)=\begin{bmatrix} f_1(t) & \cdots & f_{n_m}(t) \end{bmatrix}, \quad \boldsymbol{\varphi}_c=[\boldsymbol{\varphi}_1\boldsymbol{\varphi}_2\cdots\boldsymbol{\varphi}_{n_m}],$$
$$\boldsymbol{R}=\mathrm{diag}\begin{pmatrix} r_1 & \cdots & r_{n_m} \end{pmatrix} \tag{14}$$