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A production inventory model with fuzzy production and demand using fuzzy differential equation: An interval compared genetic algorithm approach

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ABSTRACT

In this paper, a production inventory model, specially for a newly launched product, is developed incorporating fuzzy production rate in an imperfect production process. Produced defective units are repaired and are sold as fresh units. It is assumed that demand coefficients and lifetime of the product are also fuzzy in nature. To boost the demand, manufacturer offers a fixed price discount period at the beginning of each cycle. Demand also depends on unit selling price. As production rate and demand are fuzzy, the model is formulated using fuzzy differential equation and the corresponding inventory costs and components are calculated using fuzzy Riemann-integration. α-cut of total profit from the planning horizon is obtained. A modified Genetic Algorithm (GA) with varying population size is used to optimize the profit function. Fuzzy preference ordering (FPO) on intervals is used to compare the intervals in determining fitness of a solution. This algorithm is named as Interval Compared Genetic Algorithm (ICGA). The present model is also solved using real coded GA (RCGA) and Multi-objective GA (MOGA). Another approach of interval comparison–order relations of intervals (ORI) for maximization problems is also used with all the above heuristics to solve the model and results are compared with those are obtained using FPO on intervals. Numerical examples are used to illustrate the model as well as to compare the efficiency of different approaches for solving the model.

1. Introduction

Several researchers of inventory control problem developed their production inventory models with fixed production rate. But production of an item in any manufacturing organization deeply depends on efficiency, effectiveness of the system, i.e., quality of the process output, inventory turnover ratio and so many factors related to the production process, which leads to uncertainty/impreciseness in any production process. Again due to globalization of market and introduction of multinationals in different developing countries, there is a stiff competition among different companies over the globe for marketing their product. As a result, very frequently they change their product specifications with new features and names. So for these types of products, sufficient past data are not available for the estimation of important inventory parameters like demand, production rate, etc. It is very difficult to estimate these parameters as random numbers because estimation of a random parameter requires sufficient amount of past data. As fuzzy estimations are made using experts' opinion, it is better to estimate parameters like production and demand coefficients using fuzzy numbers to reduce the error. Although a considerable number of research papers have already been published incorporating imprecise inventory parameters (Wee et al., 2009; Ryu and Yücesan, 2010; Maiti, 2011), none has considered fuzzy production rate in any production inventory model. But it is more appropriate to represent a manufacturing system. Keeping in mind the above-mentioned factor, here attention has been paid to develop an economic production quantity (EPQ) model incorporating fuzzy production rate.

Demand has been always one of the most effective factors in the decisions relating to economic ordered quantity (EOQ) model as well as EPQ model. Due to this reason, various formations of consumption tendency have been studied by inventory control practitioners, such as constant demand (Wee et al., 2009), selling price dependent demand (Ouyang et al., 2009), advertisement dependent demand (Maiti and Maiti, 2006), customer credit period dependent demand (Jaggi et al., 2008; Maiti, 2011), seasonal demand (Banerjee and Sharma, 2010), etc. Recently, You et al. (2010) developed an inventory model incorporating trial period

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dependent demand. All of them developed their models in crisp environment, i.e., demand coefficients are considered as crisp number. But, as discussed earlier, it is better to estimate demand coefficients with fuzzy numbers. In the present market situation, it is observed that some manufacturers offer price discount, specially for newly launched products for a certain time period at the beginning of each cycle. As a result, demand increases automatically due to the low unit price. After that specified period, the manufacturer withdraws the additional discount and thus unit price increases. By this process, demand increases due to the fact that some customers have already accustomed with the product during the price discount period and do not switch over to other products though price discount is withdrawn. This process of boosting a product is commonly practiced by different manufacturers specially when a product is newly launched in the market. Again, though offering of price discount boost the demand of an item, nature of demand is always fuzzy in nature. In the literature only, Pal et al. (2009) addressed a price discount inventory model. Till now none has considered price discounted fuzzy demand in an EPQ model.

The presence of fuzzy demand as well as fuzzy production rate leads to fuzzy differential equation of instantaneous state of inventory level. Till now fuzzy differential equation is little used to solve fuzzy inventory models though the topics on fuzzy differential equations have been rapidly growing in the recent years. The first impetus on solving fuzzy differential equation was made by Kandel and Byatt (1978). An extended version of their work had been published after 2 years (Kandel and Byatt 1978). After that different approaches have been made by several authors to solve fuzzy differential equations (Kaleva, 1987; Buckley and Feuring, 2000; Vorobiev and Seikkala, 2002; Chalco-Cano and Roman-Flores, 2009).

Again inventory models are normally developed with infinite lifetime for products. In reality lifetime of a product (i.e., duration of time for which demand of the item exits compare to other competitive items) rapidly changes due to several factors- innovation of new technology, introduction of new features to the item, environmental effect, etc., so planning horizon of the EPQ model of an item is finite and fuzzy/random in nature. Few research papers have already been published incorporating this assumption (cf. Pal et al., 2009; Roy et al., 2009).

In this paper, an EPQ model is presented with fuzzy production rate and fuzzy demand in an imperfect production process, i.e., not all produced units are of perfect quality. In each cycle, after the end of production process, defective units are repaired and are sold as fresh units. Demand depends on unit selling price and price discount period offered by the manufacturer cum retailer. After the discount period demand depends only on unit selling price. Also it is assumed that the planning horizon of the model is imprecise in nature, which leads to the imprecise constraint—sum of all cycle lengths is less than the length of imprecise planning horizon. For any feasible solution the constraint should hold well with at least some possibility/necessity β (Zadeh, 1978; Dubois and Prade, 1980; Liu and Iwamura, 1998). Two models are developed depending on the possibility or necessity measure of the fuzzy constraint. Fuzzy differential equation (Buckley and Feuring, 2000) and fuzzy Riemann integration (Wu, 2000) are used to develop the mathematical formulation of the model and α-cut of the total profit is derived, which is an interval. Since there is no exact method for solving an optimization problem with interval objective function, here α-cut of total profit is optimized using a heuristic. For different values of α, results are obtained and tabulated/plotted to find the nature (membership function) of the profit. Here time duration of production (t_1), discount period (t_0) and mark up of unit selling price (m_1) are decision variables. For illustration, two numerical examples are used and

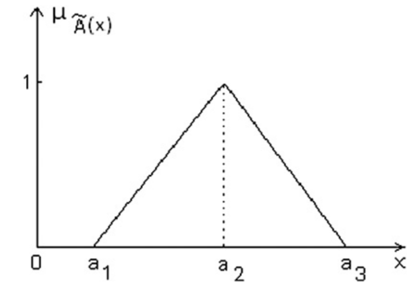

Fig. 1. Triangular fuzzy number $\widetilde{A}=(a_1,a_2,a_3)$.

results are obtained. Here, although TFN is used for fuzzy parameters, the solution methodology is quite general and can be used for and type of fuzzy number.

There are some research works on inventory control problems, where interval valued objective function is optimized. Maiti and Maiti (2006) developed an inventory model, where interval valued objective function was transformed to an equivalent multi-objective problem following an interval comparison approach (depending on left, right values of the interval numbers) proposed by Ishibuchi and Tanaka (1990) and solved using a MOGA. Gupta et al. (2009) use RBS process in a RCGA for solving an inventory model with interval valued inventory costs. They used order relations of intervals (ORI) for maximization problems (proposed by Mahato and Bhunia, 2006) for ranking the chromosomes, where center and width of intervals were used for comparison. Bera et al. (in press) solved a fuzzy inventory model, where fuzzy parameters were replaced by equivalent interval numbers (following Grzegorzewski, 2002) and objective function has been transformed to different equivalent multi-objective problems using different interval comparison approaches and solved using a MOGA. All these research papers used different approaches to compare interval objectives to find optimal decisions. Merits and demerits of different approaches on comparison of interval numbers have recently been discussed by Sengupta and Pal (2009). According to them FPO on intervals is the best approach for comparison of interval numbers. Due to this reason, in this research paper a modified GA with varying population size is used which can deal with interval objective function, where FPO on intervals is used to compare the intervals in determining fitness of a solution. This is named as ICGA and is used to solve the models. The models are also solved following RCGA (Gupta et al., 2009) and MOGA (Bera et al., in press) using FPO on intervals. At the same time ORI is also used with all the approaches- RCGA, MOGA and ICGA and results are compared with those obtained by the three approaches with FPO on intervals.

2. Preliminaries

Let $\widetilde{F_1}$ and $\widetilde{F_2}$ be two fuzzy numbers in $\Re$ with membership functions $\mu_{\widetilde{F_1}}(x)$ and $\mu_{\widetilde{F_2}}(x)$ respectively. Then according to Dubois and Prade (1980) and Zadeh (1978)

$$pos(\widetilde{F_1}*\widetilde{F_2})=\sup\{\min(\mu_{\widetilde{F_1}}(x),\mu_{\widetilde{F_2}}(y)),x,y\in\Re,x*y\} \quad (1)$$

where pos represents possibility, $*$ is any one of the relations $>,<,=,\le,\ge$ and $\Re$ represents set of real numbers

$$nes(\widetilde{F_1}*\widetilde{F_2})=1-pos(\overline{\widetilde{F_1}*\widetilde{F_2}}) \quad (2)$$

where nes represents necessity.

Similarly possibility and necessity measures of $\widetilde{F_1}$ with respect to $\widetilde{F_2}$ are denoted by $\Pi_{\widetilde{F_2}}(\widetilde{F_1})$ and $N_{\widetilde{F_2}}(\widetilde{F_1})$ respectively and are