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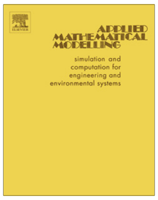

Genetic algorithm to the machine repair problem with two removable servers operating under the triadic $(0, Q, N, M)$ policy

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ABSTRACT

This paper considers the controllable M/M/2 machine repair problem with L operating machines operating under the triadic $(0, Q, N, M)$ policy. Expressions for the steady-state probabilities of the number of failed machines in the system are derived and taken in neat closed-form. We establish the total expected cost function per machine per unit time and formulate an optimization problem to find the minimum cost. The genetic algorithm (GA) is implemented to determine the optimal operating $(0, Q, N, M)$ policy and the service rate simultaneously at the global minimum cost. Sensitivity analysis with numerical illustrations is also provided.

1. Introduction

This paper deals with the optimal operation of two removable servers in the machine repair problem (MRP) with L identical operating machines operating under the triadic $(0, Q, N, M)$ policy. The working servers can be adjusted one at a time at machine's failure or service completion epochs depending on the number of failed machines in the system.

Yadin and Naor [1] first studied the N policy M/M/1 queue. They introduced the concept of an N policy which turns the server on whenever N $(N \geqq 1)$ or more customers (failed machines) are present in the system, and turns the server off when the system becomes empty. After the server is turned off, the server may not operate until N customers (failed machines) are present in the system. Bell [2] investigated the optimal operation of an M/M/2 queue with two removable servers. Rhee and Sivazlian [3] developed the busy period distribution in the controllable M/M/2 queue operating under the triadic $(0, K, N, M)$ policy. The controllable M/M/2 queue operating under the triadic $(0, Q, N, M)$ policy can generalize the ordinary M/M/2 queue, the N policy M/M/1 queue, and the ordinary M/M/1 queue. Wang and Wang [4] investigated the optimal control of an M/M/2 queue system with finite capacity L operating under the triadic $(0, Q, N, M)$ policy. Wang and Chang [5] proposed the reliability analysis of a controllable M/M/2 system with warm standbys operating under the triadic $(0, Q, N, M)$ policy. Lin and Ke [6] examined an infinite capacity multi-server system operating under the triadic policy. Recently, Lin and Ke [7] analyzed an M/M/r queueing system with infinity capacity in which the number of working servers changes depending on the queue length. They used the genetic algorithm (GA) to determine the optimal thresholds of the queue length and the corresponding service rate.

The definition of the triadic $(0, Q, N, M)$ policy is described in the following. Whenever there are no failed machines in the system, both servers are turned off temporarily, and may not reactivate until certain conditions are satisfied. Initially, we

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suppose that both servers are turned off. When the number of failed machines in the system waiting for service reaches a specific quantity N which is a decision variable, one of the two servers will be active instantly. At a later time, when the number of failed machines waiting for service increases to another specific quantity M, where $N < M$, then the remaining server will also be active instantly. However, the number of failed machines in the system decreases to Q where $3 \leqq Q < N$, while both servers are active simultaneously, the server just finishing a service will be removed from the system at that time. Furthermore, the number of failed machines in the system reaches to zero while one server is active, the server is turned off until the above conditions are met. In this article, the triadic $(0, Q, N, M)$ policy is first applied to the finite source MRP.

The genetic algorithm (GA) developed by Holland [8] and further studied by Goldberg [9], is one of the nature-inspired meta-heuristics utilized to solve difficult optimization problems. GA is a heuristic search technique based on the mechanism of natural selection and survival of fittest. It is a powerful and broadly applicable stochastic search technique for many complicated problems which are very difficult to solve by traditional techniques. Many researchers have found that GA has the superior ability of finding the approximate optimal solution compared with other heuristic algorithms such as tabu search algorithm, simulated annealing and ant colony optimization in some situations. This technique has been successfully applied to a variety of fields. Many well-known applications of GA can be found in the literature (see Lin and Ke [6,7]; Michalewicz [10]; Gen and Cheng [11]; Goldberg and Sastry [12]). Therefore, the GA has high potential for analyzing complex MRP including economic performance. In this article, the employment of GA to solve the optimal solutions for cost function with three discrete and one continuous decision variables is a new application for the optimization issue of MRP.

The main objectives of this paper are described as follows:

(1) We use a recursive method to develop the analytic steady-state solutions for the controllable M/M/2 MRP operating under the triadic (0, Q, N, M) policy.
(2) We formulate a cost management problem and use the genetic algorithm (GA) to determine the optimal operating $(0, Q, N, M)$ policy and the service rate simultaneously at the global minimum cost.
(3) We perform a sensitivity analysis to study the effects on the optimal value (Q^*, N^*, M^*, μ^*) if the system and cost parameters take on other specific values.

2. Problem statement

We consider a machine repair model with L identical operating machines that are subject to breakdowns, and are maintained by two removable servers. Each of the operating machines fails independently of the state of the others with failure rate λ. Whenever an operating machine fails, it is immediately sent to a server and is served in order of its breakdowns, with identical service rate μ. The server can serve only one failed machine at a time. Failure and service time distributions of the machines are assumed to be exponentially.

Initially, we suppose that both servers are inactive. When the number of failed machines waiting for service reaches a decision quantity N, one of the two servers activates immediately. At a later time when the number of failed machines in the system reaches another decision quantity M ($N < M$), then the remaining server will activate immediately. However, if the number of failed machines in the system decreases to a decision quantity Q ($3 \leqq Q < N$) while both servers are active simultaneously, the server just finishing a service will be removed from the system. Furthermore, if the number of failed machines reaches zero while one server is working, the server is removed from the system until the above conditions are met.

The queueing system studied with triadic policy could appear in the injection therapy room in a hospital. For example, there are two nurses (servers) operate the chemotherapy clinic, which supports the patients from breast thyroid surgery, with L (e.g., $L = 15$) injection seats. The L injection seats can be considered as L operating machines. As a patient arrives to the chemotherapy clinic, the nurse leads the patient to take an injection seat for the chemotherapy. This situation, a seat with a patient waiting for service, can be considered as a failed machine requiring the server to repair it. Due to the fact that the chemotherapy is a violent treatment to the patient, the patient usually feels nervous, scared, and depressed. Therefore, the nurse has not to start the chemotherapy when only one patient is waiting in the chemotherapy clinic to avoid over frightened for a patient. In place of this, the nurse will wait for the other arriving patients until the number of patient is over N and then start the chemotherapy. The main advantage for starting the chemotherapy with N patients is to reduce the effect of unstable atmosphere for the patient. In the chemotherapy clinic, one nurse serves the patient while the other nurse behind the reception desk is busying herself in medical preparation, inspecting patient documents, and other duties. At a later time, when the number of patients waiting for service increases to the specific quantity M, the other nurse will leave the reception desk temporarily and joint to the injection service to reduce the waiting time for patients. However, when the number of patients in the clinic decreases to Q, one nurse will remove from the patient service and return to the reception desk to prepare documents and notify the family member of the patient for the following process.

3. Steady-state results

For the controllable M/M/2 MRP operating under the triadic $(0, Q, N, M)$ policy, we describe the states of the system by the pairs (i, n), $i = 0, 1, 2$, $n = 0, 1, 2, \ldots, Q \ldots, N, \ldots, M, \ldots, L$, where $i = 0$ denotes that the server is turned off, $i = 1$, denotes that one

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