Int. J. Production Economics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A biased random key genetic algorithm for 2D and 3D bin packing problems ☆

José Fernando Gonçalves [a,*], Mauricio G.C. Resende [b]

[a] LIAAD, INESC TEC, Faculdade de Economia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-464 Porto, Portugal
[b] Algorithms and Optimization Research Department, AT&T Labs Research, 180 Park Avenue, Room C241, Florham Park, NJ 07932, USA

ARTICLE INFO

Article history:
Received 29 February 2012
Accepted 10 April 2013

Keywords:
Bin packing
Genetic algorithm
Three-dimensional
Random keys

ABSTRACT

In this paper we present a novel biased random-key genetic algorithm (*BRKGA*) for 2D and 3D bin packing problems. The approach uses a maximal-space representation to manage the free spaces in the bins. The proposed algorithm hybridizes a novel placement procedure with a genetic algorithm based on random keys. The *BRKGA* is used to evolve the order in which the boxes are packed into the bins and the parameters used by the placement procedure. Two new placement heuristics are used to determine the bin and the free maximal space where each box is placed. A novel fitness function that improves significantly the solution quality is also developed. The new approach is extensively tested on 858 problem instances and compared with other approaches published in the literature. The computational experiment results demonstrate that the new approach consistently equals or outperforms the other approaches and the statistical analysis confirms that the approach is significantly better than all the other approaches.

1. Introduction

The three-dimensional bin packing problem (3D-BPP) consists in packing, with no overlapping, a set of three-dimensional rectangular shaped boxes (items) into the minimum number of three-dimensional rectangular shaped bins (containers). All the bins have identical known dimensions (D, W, H) and each box i has dimensions (d_i, w_i, h_i) for $i = 1, \ldots, n$. Without loss of generality one can assume that all input data are positive integers and that $d_i \le D$, $w_i \le W$, and $h_i \le H$ for $i = 1, \ldots, n$. It is assumed that the boxes can be rotated. Fig. 1 shows an example of a bin packing problem with two bins and more than two hundred boxes. The two-dimensional bin packing problem (2B-BPP) addresses the problem for two-dimensional bins (W,H) and boxes (w_i, h_i) and can be treated as a special case of 3D-BPP when $d_i = D$ for $i = 1, \ldots, n$. According to the typology for cutting and packing problems proposed by Wäscher et al. (2007) bin packing problems can be classified as Single Stock-Size Cutting Stock Problem (SSSCSP) for weakly heterogeneous item sets or as 3D-SBSBPP (3D-Single Bin-Size Bin Packing Problems) for strongly heterogeneous item sets. The bin packing problem addressed in this paper is classified as 3D-SBSBPP (3D-Single Bin-Size Bin Packing Problems). The 2D-BPP and 3D-BPP are strongly NP-hard as they generalize the strongly NP-hard one-dimensional bin packing problem (Martello et al., 2000).

Three-dimensional packing problems have numerous relevant industrial applications such as loading cargo into vehicles, containers or pallets, or in packaging design. The 3D-BPP can also arise as a sub-problem of other complex problems not only in packing and cutting but also in some scheduling problems (Park et al., 1996; Hartmann, 2000).

An exact method for the 3D-SBSBPP that uses a two-level Branch & Bound method was proposed by Martello et al. (2000). Initially their proposal only solved robot-packable problems (den Boef et al., 2005), but later it was modified for solving the general problem (Martello et al., 2007). Fekete and Schepers (1997, 2004) define an implicit representation of the packing by means of Interval Graphs (*IGs*), the Packing Class (*PC*) representation. The authors consider the relative position of the boxes in a feasible packing and, from the projection of the items on each orthogonal axis, they define a graph describing the overlappings of the items in the container.

A new class of lower bounds was introduced by Fekete and Schepers (1997). The authors extend the use of dual feasible functions, first introduced by Johnson (1973), to two- and three-dimensional packing problems, including 3D-SBSBPP. Boschetti (2004) proposed the most recent lower bound, which introduces new dual feasible functions. This new bound dominates previous ones. Boschetti and Mingozzi (2003a, 2003b) propose new lower bounds for the two-dimensional case.

Several constructive and meta-heuristic algorithms have been designed for solving large bin packing problems. Faroe et al. (2003) proposed a Guided Local Search heuristic for 3D-SBSBPP and 2D-SBSBPP, based on the iterative solution of constraint satisfaction

☆Supported by Fundação para a Ciência e Tecnologia (FCT) project PTDC/EGE-GES/117692/2010.

* Corresponding author. Tel.: +351 937061379.
E-mail addresses: jfgoncal@fep.up.pt, jfgoncal@gmail.com (J.F. Gonçalves), mgcr@research.att.com (M.G.C. Resende).

0925-5273/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2013.04.019

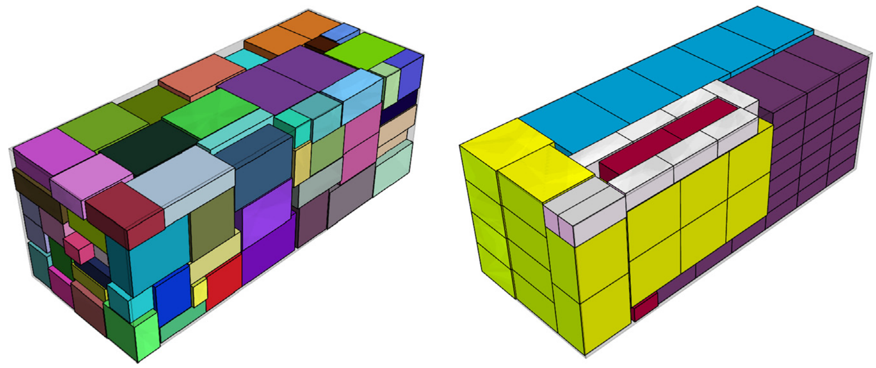

Fig. 1. Example of a bin packing problem with two bins.

problems. Starting with an upper bound on the number of bins obtained by a greedy heuristic, the algorithm iteratively decreases the number of bins, each time searching for a feasible packing of the boxes using the GLS method. Lodi et al. (1999, 2002) have developed tabu search algorithms based on new constructive procedures for two-dimensional and three-dimensional cases and in Lodi et al. (2004) propose a unified tabu search code for general multi-dimensional bin packing problems. More recently, Crainic et al. (2009) developed a two-level tabu search algorithm, using the representation proposed for nD-SBSBPP by Fekete and Schepers (2004) and Fekete et al. (2007), in which the first level aims to reduce the number of bins and the second optimizes the packing of the bins.

For the two-dimensional bin packing problem (2D-SBSBPP), Boschetti and Mingozzi (2003b) developed an effective constructive heuristic that assigns a score to each box, considers the boxes according to decreasing values of the corresponding scores, updates the scores using a specified criterion, and iterates until either an optimal solution is found or a maximum number of iterations is reached. Monaci and Toth (2006) designed a set-covering-based heuristic approach in which in a first phase a large number of columns are generated by heuristic procedures and by the execution of the exact algorithm by Martello and Vigo (1998) with a time-limit. In the second phase these columns are used for solving a set-covering problem which gives the solution to the original bin packing problem. Parreño et al. (2010) propose a new hybrid GRASP/VND algorithm for solving the 3D-SBSBPP bin packing problem which can also be directly applied to the two-dimensional case (2D-SBSBPP). The constructive phase is based on a maximal-space heuristic developed for the container loading problem. In the improvement phase, several new moves are designed and combined in a VND structure. Mack and Bortfeldt (2012) present a straightforward heuristic for the 3D-SSSCSP where all items may be rotated and the guillotine cut constraint has to be respected. The heuristic is based on a method for the container loading problem following a wall-building approach and on a method for the one-dimensional BPP.

3D-SBSBPP is NP-hard in the strong sense. Therefore, when large instances are considered, heuristics are the methods of choice. In this paper we present a novel biased random-key genetic algorithm (BRKGA) for the 2D-SBSBPP and 3D-SBSBPP. The approach uses a maximal-space representation to manage the free spaces in the bins. The proposed algorithm hybridizes a novel placement procedure with a genetic algorithm based on random keys. The BRKGA is used to evolve the order in which the boxes are packed into the bins and the parameters used by the placement procedure.

The remainder of the paper is organized as follows. In Section 2 we introduce the new approach, describing in detail the BRKGA, the novel placement strategy, the novel fitness function, and the parallel implementation. Finally, in Section 3, we report on computational experiments, and in Section 4 make concluding remarks.

2. Biased random-key genetic algorithm

We begin this section with an overview of the proposed solution process. This is followed by a discussion of the biased random-key genetic algorithm, including detailed descriptions of the solution encoding and decoding, evolutionary process, fitness function, and parallel implementation.

2.1. Overview

The new approach is based on a constructive heuristic algorithm which places the boxes one at a time in the bins. A new bin is opened when the box that we are trying to place does not fit in the bins that are already open (note that all the bins stay open until all boxes are packed). The management of the feasible placement positions is based on a list of empty *maximal-spaces* as described in Lai and Chan (1997). A 2D or 3D empty space is maximal if it is not contained in any other space in the bin. Each time a box is placed in an empty maximal-space, new empty maximal-spaces are generated. The new approach proposed in this paper combines a biased random-key genetic algorithm, a new placement strategy, and a novel fitness function.

The role of the genetic algorithm is to evolve the encoded solutions, or *chromosomes*, which represent the *box packing sequence* (*BPS*) and the vector of box orientations (*VBO*) used for packing the boxes into the bins. For each chromosome, the following phases are applied to decode the chromosome:

(1) *Decoding of the box packing sequence*: This first phase decodes part of the chromosome into the *BPS*, i.e. the sequence in which the boxes are packed into the bins.
(2) *Decoding of box orientations*: The second phase decodes part of the chromosome into the vector of box orientations *VBO* to be used by the placement procedure.
(3) *Placement strategy*: The third phase makes use of *BPS* and *VBO*, defined in phases 1 and 2, and constructs a packing of the boxes into the bins.
(4) *Fitness evaluation*: The final phase computes the fitness of the solution (or measure of quality of the bin packing). For this phase we developed a novel measure of fitness which improves the quality of the solutions significantly.

Fig. 2 illustrates the sequence of steps applied to each chromosome generated by the BRKGA.

Please cite this article as: Gonçalves, J.F., Resende, M.G.C. A biased random key genetic algorithm for 2D and 3D bin packing problems. International Journal of Production Economics (2013), http://dx.doi.org/10.1016/j.ijpe.2013.04.019