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# Hybrid genetic algorithm approach for precedence-constrained sequencing problem

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ABSTRACT

The objective of precedence-constrained sequencing problem (PCSP) is to locate the optimal sequence with the shortest traveling time among all feasible sequences. Various methods for effectively solving the PCSP have been suggested. This paper proposes a new concept of hybrid genetic algorithm (HGA) with adaptive local search scheme in order that the PCSP should be effectively solved. By the use of the adaptive local search scheme, the local search is automatically adapted into the loop of genetic algorithm. Two types of the PCSP are presented and analyzed to compare the efficiency among the proposed HGA approach and other competing conventional approaches. Finally, it is proved that the proposed HGA approach outperforms the other competing conventional approaches.



## 1. Introduction

The objective of precedence-constrained sequencing problem (PCSP) is to locate the optimal sequence with the shortest traveling time through all nodes, visiting each node only once. One of the important problems in the PCSP is to consider the precedence relation between nodes *i* and *j*. The precedence relation determines the sequence of travel between the given pair of nodes, *i* and *j*; i.e., *i* must be visited before *j*. Therefore, this problem can be formulated as a traveling salesman problem (TSP) with precedence constraints. Locating an optimal sequence for the PCSP may cause a great effect, since some inefficient sequences that may be generated in the PCSP highly affect all downstream stages such as manufacturing, logistics, and networking.

Nevertheless, the PCSP has been successfully applied to a number of optimization problems regarding networks, scheduling, project management, logistics, assembly flow, and routing. We can classify the studies related with the PCSP into two types. First type is to use various heuristics and second type applies artificial intelligent algorithms such as genetic algorithm (GA) in order to solve the various types of the PCSP.

For first type, He and Kusiak (1992) proposed a heuristic algorithm to solve the PCSP with sequence-dependent changeover cost and precedence constraints. Savelsbergh and Sol (1995) suggested a heuristic algorithm to solve the Dial-A-Ride problem that a vehicle should transport a number of passengers, and each passenger should be transported from a given location to a specific destination point. The Dial-A-Ride problem can be displayed by a PCSP. Renaud, Boctor, and Ouenniche (2000) suggested a heuristic model that solves the pickup and delivery traveling salesman problem (TSP) which can be represented by a PCSP. Duman and Or (2004) developed a heuristic approach to solve a PCSP in a printed circuit board assembly problem. This approach initially ignores precedence relations and solves the problem as a pure TSP, and then it is applied to eliminate component damage in the resulting TSP tour.

For second type, Moon, Kim, Choi, and Seo (2002) presented a GA approach with a priority-based representation procedure to solve the PCSP based on supply chain planning model. Chan and Chung (2004) proposed a multi-objective GA procedure for the order distribution problem in a demand driven logistics which can be represented by a PCSP. Altiparmak, Gen, Lin, and Paksoy (2006) and Altiparmak, Gen, Lin, and Karaoglan (2007) developed multi-objective GA approach for a single-source, single and multi-product, multi-stage supply chain network design problem. This problem has various precedence constraints and is thus presented as a PCSP. Gen, Cheng, and Lin (2008) showed various types of network models with precedence constraints and the models were presented as a priority-based representation procedure in GA implementation. Gen, Lin, and Zhang (2009) suggested a network model with various pickup and delivery points. This model was displayed as a PCSP and solved by GA approach. Recently, Yun and Moon (2011) proposed a GA approach to solve various types of the PCSP. The proposed GA approach used topological sort (TS)-based representation procedure to effectively represent the PCSP.

The conventional approaches mentioned above show that the PCSP is a useful tool for representing various types of industrial

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optimization problems with precedence constraints (Chen, 1990; Duman & Or, 2004; Gen et al., 2008; Gen et al., 2009; He & Kusiak, 1992; Moon et al., 2002).

However, there are some difficulties in modeling the PCSP. First, most of the conventional approaches may not solve the PCSPs efficiently because of its computational complexities and complicated precedence constraints. Secondly, computation time for locating the optimal solution of the PCSP highly increases with the increase of the number of nodes.

Therefore, a new approach is required to overcome these difficulties. The new approach can easily handle complicated precedence constraints and effectively finds an optimal solution or sequence in the PCSP regardless of the increase of the number of nodes.

This paper aims at formulating a mathematical model as well as developing a new hybrid GA (HGA) approach for effectively solving various types of the PCSP. The proposed mathematical model is formulated by modifying the standard integer programming for TSP in order to avoid computational difficulties and ambiguity. In the HGA approach, the TS-based representation procedure is used to generate a set of feasible sequences, which is used for producing the individuals of the population in the proposed HGA approach. An adaptive local search scheme is used for adaptively regulating the use of local search in the proposed HGA approach.

## 2. Modeling

Main characteristic of the PCSP is that there is no cycle in its precedence relations. It is clear that a linear sequence is impossible if the given direct graph for the PCSP has a loop, because a sequence from nodes $i$ to $j$ and from nodes $j$ to $i$ is possible for two nodes $i$ and $j$ on the cycle.

A feasible sequence passes through each node in the given directed graph only once and cannot visit any two nodes at the same time. Therefore, the PCSP is a type of directed acyclic graph. Since most of directed graphs have some feasible sequences, it is necessary to find the optimal sequence among all the feasible ones. The optimal sequence, which minimize the total traveling time, can be decided by comparing all feasible sequences.

Various constraints for treating precedence relations at each node should be also considered for implementing the objective of the PCSP. The following notation is used for modeling of the PCSP.

- $i, j$ node index, $i, j = 1, \ldots, I$, where $I$ is the number of nodes.
- $TT_{ij}$ traveling time from nodes $i$ to $j$.
- $AT_i$ arrival time at node $i$.
- $S1$ set of nodes $(i, j)$, where node $i$ is visited before node $j$.
- $S2$ set of nodes $(i, j)$, where nodes $i$ and $j$ can be visited in any feasible sequence.
- $LP$ arbitrary large positive number.

The variable is introduced to adapt the PCSP as follows:

$$x_{ij} = \begin{cases} 1, & \text{if node } i \text{ is visited before } j, \\ 0, & \text{otherwise.} \end{cases}$$

Let $E$ be the finding a node with the maximal arrival time. The $E$ can be represented as follows:

$$E = \max_{\forall i}\{AT_i\} \tag{1}$$

Finally, a mixed-integer programming model of the PCSP for minimizing the total traveling time is as follow (Yun & Moon, 2011):

Minimize

$$E = \max_{\forall i}\{AT_i\} \tag{2}$$

$$\text{subject to } AT_j - AT_i \geq TT_{ij} \qquad \forall (i,j) \in S1 \text{ and } i \neq j \tag{3}$$

$$AT_i - AT_j + LPx_{ij} \geq TT_{ij}x_{ij} + TT_{ji}x_{ji} \qquad \forall (i,j) \in S2 \text{ and } i \neq j \tag{4}$$

$$X_{ij} + x_{ji} = 1 \qquad \forall (i,j) \in S2 \text{ and } i \neq j \tag{5}$$

$$X_{ij} = 0, 1 \qquad \forall (i,j) \in S2 \text{ and } i \neq j \tag{6}$$

Constraints (3) and (4) ensure that each two node cannot be visited at the same time. Constraint (5) represents that any two nodes are to be visited in only one sequence. This constraint means the technical restriction on visiting sequence. Therefore a feasible sequence among the nodes with precedence relations is existed. Also, this constraint represents that there is no cycles in precedence relations. Constraint (6) restricts all variables to integer values, 0 or 1.

## 3. HGA approach

The detailed procedures of implementing the proposed HGA approach appear in the following subsections. First, a TS-based representation procedure is suggested for effectively treating the precedence constraints. By using the TS-based representation procedure, the initial population of the HGA approach is made. Secondly, a fitness test and a selection procedure for the individuals resulting from the proposed HGA search process are undertaken. Third, crossover and mutation operators are used for genetic operators. Lastly, an adaptive local search scheme is applied to determine the use of local search in the HGA loop.

### 3.1. Representation and Initialization

Of the well-known methods for treating precedence constraints in the PCSP, the priority-based representation procedure has been developed by several researchers (Gen, Altiparmak, & Lin, 2006; Gen & Cheng, 2000; Moon & Seo, 2005; Moon et al., 2002). The priority-based representation procedure has been widely used in representing the feasible sequences with precedence constraints. Its key issue is to assign randomly generated priorities to each node. Therefore, the selection of each node always relies on the priority assigned. Finally, various types of feasible sequences may not be produced because of the priority constraint that is assigned to each node, which is called idling and may restrict the efficiency of the search process. In order to avoid these difficulties when using the priority-based representation procedure, we use the TS-based representation procedure (Yun & Moon, 2011). This procedure can effectively treat precedence constraints in the PCSP.

In general, a precedence relation between nodes $i$ and $j$ is represented as predecessor or successor of each node in any sequence. If they visit each other, this situation is called as cycle. To represent precedence relations in the PCSP, a precedence matrix $D = [d_{ij}]$ is used, where

$$d_{ij} = \begin{cases} 1, & \text{if node } i \text{ is visited immediatelybefore } j, \\ \text{null}, & \text{otherwise.} \end{cases}$$

where $d_{ij} = 1$ indicates the precedence relation $\langle i, j \rangle$.

The precedence matrix of the example with eight nodes and the precedence relations is shown in Table 1.

For example, $d_{24} = 1$ in Table 1 means that node 2 should be visited immediately before node 4, that is, node 2 precedes node 4 in linear sequence. On the other hand, if $d_{ij}$ = null, then there is no relationship between two nodes i and j. The columns 1 and 2 in the precedence matrix have 'null' values, which indicates that nodes 1 and 2 have no predecessors. From the precedence relations in Table 1, we can draw the directed graph shown in Fig. 1.