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A rough penalty genetic algorithm for constrained optimization

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ABSTRACT

Many real-world issues can be formulated as constrained optimization problems and solved using evolutionary algorithms with penalty functions. To effectively handle constraints, this study hybridizes a novel genetic algorithm with the rough set theory, called the rough penalty genetic algorithm (RPGA), with the aim to effectively achieve robust solutions and resolve constrained optimization problems. An infeasible solution is subjected to rough penalties according to its constraint violations. The crossover operation in the genetic algorithm incorporates a novel therapeutic approach and a parameter tuning policy to enhance evolutionary performance. The RPGA is evaluated on eleven benchmark problems and compared with several state-of-the-art algorithms in terms of solution accuracy and robustness. The performance analyses show this approach is a self-adaptive method for penalty adjustment. Remarkably, the method can address a variety of constrained optimization problems even though the initial population includes infeasible solutions.

1. Introduction

Most real-world optimization problems involve constraints. Since Holland first developed a genetic algorithm (GA) in 1975 [13], GAs have been successfully applied to a wide range of complex problems in science, engineering, and industry fields. The challenge of a constrained problem is how to optimize the objective function value against its constraint violations. However, traditional GAs may consume considerable computational energy in searching infeasible solutions because genetic operations do not always preserve feasibility [28]. Considerable research has focused on constraint-handling techniques for evolutionary algorithms (EAs) [37,38].

Traditionally, penalty-function methods are the most popular constraint-handling techniques [22]. Each infeasible solution is penalized by the magnitude of its constraint violations. Many studies attempt to manipulate penalty coefficients for balancing the objective function with constraint violations [4,7]. However, the static penalty method has difficulty in adjusting all the penalty factors empirically [14]. Although the dynamic penalty (DP) method uses evolutionary time to compute the corresponding penalty factors, it is still difficult to determine dynamic penalty functions appropriately [15]. The adaptive penalty (AP) method considers the feasibility ratios of sequential generations to determine penalties [8]. Nevertheless, the drawback of the AP method is the need to choose generational gap and coefficient adjustments.

This study attempts to answer the following questions: (1) Which constraints dominate an optimization problem? (2) What kind of fitness metrics can evaluate infeasible solutions in a meaningful way? (3) How can parameters be adjusted appropriately? Therefore, a novel penalty-adjustment method is proposed in this paper to guide genetic evolution for approaching the global optimal solution. Because the proposed method is inspired by Pawlak's rough set theory (RST) [24,25], it is named the rough penalty (RP) method, which serves as a new constraint-handling technique that aims to

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effectively apply the information granulation technique of RST in dealing with indiscernibility penalties. In this paper, the RP method is further incorporated with a GA, named RPGA, for solving constrained optimization problems with the following two goals: (1) adjust penalty coefficients according to their constraint violations and (2) analyze inefficient constraints by applying RST during evolution. The advantage of the proposed RPGA is that it can automatically adjust penalty coefficients for each optimization problem. Furthermore, the method does not require extra functional analysis to realize its solution space. The performance of the RPGA is evaluated by eleven well-known constrained problems. Experimental results show the proposed RPGA cannot only find optimal or close-to-optimal solutions but also obtain robust results for both linear and nonlinear constraint functions.

The remainder of this paper is organized as follows. Section 2 briefly describes several constraint-handling techniques. Section 3 introduces the proposed RST-based penalty function. Next, Section 4 describes the proposed RPGA and its genetic operations. In Section 5, we adopt Taguchi's quality-design method to analyze the best parameter setting for the proposed RPGA. Section 6 reports the computational results for 11 constrained optimization problems and comparisons with several well-known optimization algorithms. Finally, we summarize the findings and contributions of this study in Section 7.

2. Constraint handling techniques in evolutionary algorithms

Without a loss of generality, a general minimization problem with m constraints can be formulated as [30]

$$\text{minimize} \quad f(\vec{x}) = f(x_1, x_2, \ldots, x_n) \tag{1}$$

$$\text{subject to} \quad g_k(\vec{x}) \leqslant 0, \quad k = 1, \ldots, q \tag{2}$$

$$h_k(\vec{x}) = 0, \quad k = q+1, \ldots, m \tag{3}$$

$$\overrightarrow{B}_l \leqslant \vec{x} \leqslant \overrightarrow{B}_u, \tag{4}$$

where $f(\vec{x})$ is the objective function, $\vec{x} = (x_1, x_2, \ldots, x_n)$ is a vector of n decision variables, and $\overrightarrow{B}_l$ and $\overrightarrow{B}_u$ represent the lower bounds (LBs) $(B_{l1}, B_{l2}, \ldots, B_{ln})$ and the upper bounds (UBs) $(B_{u1}, B_{u2}, \ldots, B_{un})$ of all the variables, respectively. There are q inequality constraints $g_k(\vec{x})$ and $(m-q)$ equality constraints $h_k(\vec{x})$. A constrained optimization problem can be transformed into an unconstrained one by introducing penalty terms into the original objective function such that it becomes an extended objective function as in Eq. (5).

$$\psi(\vec{x}) = f(\vec{x}) + \sum_{k=1}^{m} (w_{tk} \times \max(0, \Phi_k(\vec{x}))^2), \tag{5}$$

where $\psi(\vec{x})$ is the expanded objective function. Penalty coefficients w_{tk} are adaptive with the magnitude of each constraint violation for the kth constraint in the tth generation. For a minimization problem, let

$$\Phi_k(\vec{x}) = \begin{cases} g_k(\vec{x}), & k = 1, \ldots, q \\ |h_k(\vec{x})| - \delta, & k = q+1, \ldots, m \end{cases} \tag{6}$$

where "$|\cdot|$" denotes an absolute operator, and the small tolerance (δ) is assigned to 0.0001 for equality constraints.

Generally, penalty-function approaches attempt to optimize the expended objective function and search the boundary between feasible and infeasible regions [7]. As infeasible solutions are penalized by Eq. (5), the search ability of EAs is significantly influenced by the values of the penalty terms [23]. A large penalty will reduce the net fitness of infeasible solutions. That discourages an EA to explore the infeasible region; thus, the EA may ignore solutions near the boundary of the feasible region. A low penalty misdirects an EA to explore the infeasible region because the penalty will be negligible with respect to its objective function [3]. Therefore, penalty coefficients should be adjusted according to the evolutionary situation [18,32].

Barbosa and Lemonge (denoted as BL) in 2003 [1] and Lemonge and Barbosa (abbreviated as LB) in 2004 [19] proposed two AP-based functions that assign different penalties to different constraints according to the average value of the objective function and the level of each constraint violation. Tessema and Yen (represented as TY) in 2006 [34] introduced a distance-based fitness function to the normalized fitness-constraint violation space. Two penalty values are applied to infeasible individuals to identify promising infeasible individuals and guide the search process toward finding the optimal solution. Farmani and Wright (indicated as FW) in 2003 [6] developed a two-stage penalty method in which the infeasibility values of an infeasibility solution are represented by its relationship to the worst, the best, and the highest values of the objective function in the current population. Although this method produced good results for most test functions, this two-stage penalty method has an unnecessarily high computational cost.

In this paper, the proposed RP method, which benefits from the concept of the RST and DP method, is utilized to adjust penalties based on their constraint violations. This proposed RPGA can achieve two primary advantages. First, parameter tuning in this work is self-adaptive. Second, the RPGA can obtain the global optimum starting from any infeasible solution.

3. Rough penalizing method

The rough set theory (RST) proposed by Pawlak in 1982 [24] is a mathematical approach for dealing with vagueness in which imprecision is expressed by a boundary region of solution space and imperfect searching is approximated by the main