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A note on the random yield from the perspective of the supply chain

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ABSTRACT

Keren [The single-period inventory problem: extension to random yield from the perspective of the supply chain. Omega 2009;37:801–10] considers a supply chain in which the distributor faces a known demand and orders from the producer subject to a random production yield, and shows that the distributor may find it optimal to order more than what is needed due to supply uncertainty under a uniform distribution. However, Keren (2009) does not address the questions whether it is always optimal for the distributor to order more, or when to order more. In this note, we point out that ordering more is not always an optimal strategy and specify the condition under which this strategy becomes optimal. We also examine the profit losses of the supply chain members resulting from the random yield supply, which is another question not considered in Keren (2009). The producer is found to possibly benefit from this production yield uncertainty, although the performances of the distributor and of the entire supply chain are always undermined. Our results are obtained under a more generalized yield distribution, and can thus be applied to wider industrial domains.

1. Introduction

A recent paper by Keren [1] analyzes a two-tier supply chain, in which a distributor facing a deterministic demand procures a product from a producer confronting a random production yield under the wholesale price contract. An analytical solution to the distributor's ordering decision is derived when the production yield follows the uniform distribution, and the optimal ordering quantity is shown possibly more than the known demand through numerical examples. In this note, we revisit this problem and obtain new results by further examining supply chain decisions and profits for the generalized distribution of yield randomness. Our contribution mainly lies in the following areas:

(1) We derive analytical solutions to the optimal decisions for supply chain members, and provide explicit conditions under which the distributor should order more than the demand. These conditions are found relevant in different ways to the yield distribution for the cases of additive risk and multiplicative risk, which indicates the importance of recognizing the type of production yield risk. These results surpass those of Keren [1].

(2) We derive analytical solutions of the profit losses for supply chain members due to the random production yield, and verify that the performances of the distributor and the entire supply chain are always worse off. However, the producer could benefit from this random yield under certain conditions, which indicates the importance of deriving a more effective risk-sharing mechanism rather than the simple wholesale price scheme. This issue is not addressed by Keren [1].

(3) All of our results are based on the generalized distribution of yield randomness, thus bearing significance in both theoretical and practical domains. In contrast, Keren [1] only provides analytical solutions for the uniform distribution.

The problem in this paper stands on the research interface between random yield and supply chain management, both of which have generated a large body of literature, including the studies of Gerchak et al. [2], Anupindi and Akella [3], Erdem and Ozekici [4], Gupta and Cooper [5], and Mukhopadhyay and Ma [6], among others, on production planning under random yield, and Taylor [7], Cachon and Lariviere [8], Kaya [9], Jornsten et al. [10], and Hosoda and Disney [11], among others, on decision interaction in the supply chain. Aside from the aforementioned studies, more related publications involve supply chain models under the supplier's random yield, which are briefly reviewed below. Zimmer [12] investigates the supply chain coordination issue between a buyer facing deterministic demand and a supplier facing uncertain capacity that is equal to the sum of newly built capacity and a random leftover. He and Zhang [13] consider the risk-sharing problem between a retailer and a supplier subject to random production yield and a more expensive emergency

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production option. In a subsequent work, He and Zhang [14] further explore a supply chain with random yield production and a yield-dependent secondary market, in which the product is acquired or disposed of. Serel [15] considers a supply chain in which the retailer orders products from an unreliable supplier and a reliable manufacturer, analyzing the retailer's ordering problem in conjunction with the manufacturer's pricing problem. Xu [16] investigates a supply chain model in which the retailer adopts an option contract and the supplier conforms with random yield production and a more expensive emergency procurement. In addition, a number of recent studies explore issues of supply chain coordination of an assembly system with random yield supply. Such studies include those by Gurnani and Gerchak [17], Guler and Bilgic [18], and Yan et al. [19]. The reader can also refer to Keren [1] and the references therein for more literature.

The rest of this paper is organized as follows. In Section 2 we restate the model and define the notations in use. Section 3 focuses on the optimal decisions of the supply chain members, and Section 4 explores the impacts of yield randomness on the system profits. Section 5 concludes the paper.

2. Model and notation

In this section we restate the supply chain model of Keren [1]. Consider that a distributor faces a known demand d and orders from a producer who has a random production yield. For the planned production quantity e, the producer's output quantity $S(e)$ conforms to one of the two following forms: additive risk $S(e)=e+\xi$ or multiplicative risk $S(e)=e\cdot\xi$, where ξ denotes the random disturbance in the production process with finite support $[A,B]$, density function $f(\cdot)$, and distribution function $F(\cdot)$. We assume that $f(\cdot)$ is positive with its argument. Furthermore, let $F^{-1}(\cdot)$ be the inverse function of $F(\cdot)$ and $\overline{F}(\cdot)=1-F(\cdot)$ be the tail distribution. Obviously, in the case of multiplicative risk, ξ should be defined on a positive support and thus $A\geq 0$. In the case of additive risk, we assume $d\geq B$ to rule out the trivial case that the producer makes zero production input and relies only on the additive fluctuation to satisfy demand.

The sequence of events can de described as follows: The distributor first submits an order quantity q to the producer at an exogenous wholesale price w, then the producer responsively determines the planned production quantity e. After the realization of random yield, the producer delivers the minimum of the output production quantity and the order, with the wholesale price w paid for each delivery. Other parameters include selling price p for the distributor, production cost c for the producer (incurred by each planned unit even when not converted to the final yield), as well as leftover holding costs h_1 and h_2 for the distributor and the producer (those can be negative if the leftover earns salvage value), respectively, which satisfy $c+h_2>0$, $w+h_1>0$, and $c<w<p$. Note that Keren [1] includes other parameters such as handling cost and goodwill penalties for unmet demand and orders, which are neglected in this paper for ease of exposition. Nevertheless, these parameters can be incorporated without any difficulty and all of our results remain valid after minor parameter modification. For example, if b is incorporated as the penalty for each unit of unsatisfied demand for the distributor, then all of the results in this paper still hold except that all "p" should be changed into "$p+b$".

Keren [1] proposes a framework for solving this problem and provides analytical results for the distributor's ordering quantity when yield risk ξ conforms to uniform distribution. In contrast, the solution derivation and managerial analysis in this paper are based on the generalized distribution of ξ. Without loss of generalization, we normalize its expectation into $E\xi=0$ in the additive risk case and $E\xi=1$ in the multiplicative risk case.

3. Optimal decisions

We employ the standard Stackelberg game analysis procedure by first exploring the producer's problem. (The reader can refer to Fudenberg and Tirole [20] on the details of Stackelberg game and its analysis procedure.) The producer's expected profit for a given order quantity q is

$$\pi_p(e)=E\{w\min[q,S(e)]-h_2[S(e)-q]^+-ce\}. \tag{1}$$

Lemma 1. *For any given order quantity q from the distributor, the producer's optimal production quantity is*

(1) $e^*=(q-k_1)^+$, *where* $k_1=F^{-1}((c+h_2)/(w+h_2))$, *for* $S(e)=e+\xi$;
(2) $e^*=q/k_2$, *where* k_2 *is characterized by* $\int_0^{k_2} tf(t)\,dt=(c+h_2)/(w+h_2)$, *for* $S(e)=e\cdot\xi$.

Lemma 1 characterizes the optimal production quantity of the producer, given the distributor's order. Specifically, this quantity is equal to the order minus a constant (as long as the difference is positive) for the case of additive risk and the order divided by a constant for the case of multiplicative risk. These constants are related to the distribution of yield randomness, the production and holding costs, and the wholesale price. It is clearly shown that for a given order quantity, the producer's optimal production quantity increases with the wholesale price and decreases with the producer's holding and production costs.

Taking into account the producer's response, the distributor's problem is to determine order quantity q to maximize her expected profit:

$$\pi_d(q)=E\{p\min[d,q,S(e^*(q))]-w\min[q,S(e^*(q))] -h_1[\min[q,S(e^*(q))]-d]^+\} \tag{2}$$

with $e^*(q)$ given in Lemma 1. We have the following result.

Proposition 1. *The distributor's optimal order quantity q^* and the induced producer's production quantity e^* are*

(1) *for* $S(e)=e+\xi$

$$\begin{cases} q^*=d+\max(k_1-k_3,0), \\ e^*=d-\min(k_1,k_3), \end{cases} \tag{3}$$

where $k_3=F^{-1}((w+h_1)/(p+h_1))$;

(2) *for* $S(e)=e\cdot\xi$

$$\begin{cases} q^*=d\cdot\max\left(\dfrac{k_2}{k_4},1\right), \\ e^*=d/\min(k_2,k_4), \end{cases} \tag{4}$$

where k_4 *is characterized by*

$$\int_0^{k_4} tf(t)\,dt=\frac{(c+h_2)(w+h_1)}{(p+h_1)(w+h_2)}+\left(\frac{w+h_1}{p+h_1}\right)\cdot k_2\overline{F}(k_2).$$

Proposition 1 characterizes the optimal order and production decisions of the supply chain members. As Keren [1] suggests, the distributor's optimal order quantity is always equal to or greater than the known demand. More specifically, this order quantity is related to the distribution of yield randomness and the system parameters of prices and costs. In the additive risk case, this