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Modeling the dynamics of congestion in large urban networks using the macroscopic fundamental diagram: User equilibrium, system optimum, and pricing strategies

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ABSTRACT

The macroscopic fundamental diagram (MFD) is introduced in recent studies to present the relationship between the flow and the density of the network in large urban regions (neighborhoods). The MFD can be also rescaled to approximate network outflow as a function of the vehicular accumulation of the system in the morning commute problem. In this research, we develop a bathtub model (macro-scale traffic congestion model) by combining Vickrey's (1969) model of dynamic congestion with the MFD to formulate the user equilibrium over the peak as an ordinary differential equation (ODE). This problem can be solved numerically to estimate the exact solution of the morning commute problem. Alternatively, the morning commute problem can be solved analytically by approximating the solution of the ODE using a well-behaved function. Here, we present a quadratic and also a linear approximation of the equilibrium solution for a semi-quadratic MFD, considering that the declining part of the MFD is shown to be well estimated by a quadratic function. To optimize the system, we present pricing strategies for network users (dynamic tolling) and employers inside the region (dynamic taxing) that can minimize the generalized cost of the system by keeping the outflow maximized over the peak. Finally, we compare the exact and the approximate solutions of the problem, and also the proposed pricing strategies of the region in a numerical example.

1. Introduction

Urban gridlock occurs and expands quickly in a cascade-like manner in a transportation network when continuous queues of vehicles start blocking adjacent intersections. This phenomenon can directly affect the performance of the transportation system and worsen the traffic condition over the peak. Research on the macroscopic relationship between traffic variables of the network shows that the decline in the average speed of network with the rise in the accumulation of the system becomes dramatically sharp over the peak due to the effect of gridlock. In this condition, network users experience longer travel times in their trips, while it becomes physically impossible for them to arrive to their destinations punctually. Rational users seek to minimize the generalized cost of their trips by adjusting their own trip starting time. Under the per-

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fect information assumption, the interaction between users seeking an optimal schedule for their own trips in the network leads to the user equilibrium in which no users can reduce the cost of their trip by unilaterally shifting their starting time.

The trip scheduling problem is introduced in Vickrey's (1969) congestion theory for a first-in, first-out single bottleneck with fixed capacity and uniformly-distributed demand over the peak. Vickrey's (1969, 1973) model of dynamic congestion has been further elaborated in the literature by considering a distribution for the wished departure time of the users, and also accounting for heterogeneity in both schedule penalty preferences and the user valuation of time (Henderson, 1974, 1977, 1981; Hendrickson and Kocur, 1981; Newell, 1987; Arnott et al., 1988, 1992, 1994; Van den Berg and Verhoef, 2011a, b; Xiao et al., 2011; Tian et al., 2013; Xiao et al., 2015). The morning commute problem in a single bottleneck has been also formulated as an ODE in Wu and Huang (2015) for a population of users with identical wished departure times and heterogeneous schedule deviation penalty preferences and value of time, without allowing late arrivals. Amirgholy and Gonzales (2017) also make use of the efficient frontier concept from the field of finance to propose an analytical solution and also an optimal pricing strategy for the morning commute problem with a general distribution of schedule preferences over time and a continuous joint distribution of schedule penalty preferences over the population of commuters. The existence and uniqueness of the equilibrium solution is also shown in the literature (Smith, 1984; Daganzo, 1985; Lindsey, 2004).

Congestion theory also has wide applications in modeling queuing systems with time-dependent demand and state-variable capacity (Amirgholy and Gonzales, 2016). The transportation network in large urban regions (neighborhoods) can be conceived as a macro-scale queueing system with a time-dependent inflow. In that sense, the user equilibrium in the network can be described by the congestion theory in a bathtub model, in which the effect of congestion on the performance of the system is captured by the macroscopic fundamental diagram (MFD). The idea of using the macroscopic relationship between network traffic variables for optimizing the accumulation of vehicles in an urban region is introduced in Godfrey (1900), and elaborated in Small and Chu (2003) and Daganzo (2007). With this groundwork, the MFD model was developed analytically in Daganzo and Geroliminis (2008) and measured empirically for the city of Yokohama in Geroliminis and Daganzo (2008). The macroscopic relationship between traffic variables in real urban networks can be approximated by estimating a limited number of observable parameters (Laval and Castrillon, 2015). Gan et al. (2017) also derive an analytical approximation of this relationship through a closed-form formula for the MFD in signalized urban networks. Alternatively, the MFD of the region and the average vehicle density of the network can be estimated using low-penetrated probe data from the network (Gayah and Dixit, 2013; Ji et al., 2014; Leclercq et al., 2014; Nagle and Gayah, 2014; Du et al., 2016; Ambühl and Menendez, 2016; Loder et al., 2017). Recent studies on the properties of the MFD in urban regions show that the scatter and the shape of the MFD largely depend on the spatial distribution of the vehicular density in the network (Mazloumian et al., 2010; Geroliminis and Sun, 2011; Gayah and Daganzo, 2011; Mahmassani et al., 2013). To cope with the heterogeneity in vehicular density of real-size urban regions, different models are proposed in the literature to derive well-defined (low-scattered) MFDs in multi-region urban networks, and also to design real-time perimeter flow control strategies to improve the mobility of users in the region (Ji and Geroliminis, 2012; Haddad and Geroliminis, 2012; Geroliminis et al., 2013; Aboudolas and Geroliminis, 2013; Haddad et al., al.,2013; Gayah et al., 2014; Ramezani et al., 2015; Du et al., 2015; Xie et al., 2016; Ji et al., 2016; Saeedmanesh and Geroliminis, 2016; Jiang et al., 2016; Zhong et al., 2017; Ding et al., 2017; Ampountolas et al., 2017).

The dynamics of congestion in urban networks can be modeled on a macro-level by combining Vickrey's congestion theory with the MFD. However, dependence of network outflow at each point in time upon prior states of traffic over the peak makes the problem intractable, as mentioned in Arnott (2013). To tackle such inherent intractability in the equilibrium problem, various simplifying assumptions are made in different studies to solve tractable versions of the problem. In Small and Chu (2003), the underlying assumption is that the travel time of users at each point in time just depends on the density of the system at their arrivals to the network. Geroliminis and Levinson (2009) also use the same simplifying assumption as Small and Chu (2003) by ignoring the effect during individual user trips of variations in density on their travel times in the network. A cordon pricing strategy is also proposed for the downtown area of urban regions to improve mobility in the network by eliminating delay and increasing outflow in the network, although their proposed pricing strategy is not designed to maximize outflow. To derive an equilibrium solution for the capacity-variable bottleneck problem, Fosgerau and Small (2013) simplify the MFD model by considering two levels of outflow for the network, and propose a pricing strategy that maximizes the social welfare of users by eliminating the delay in the system. Arnott (2013) tackles the problem from a different angle by assuming that the probability that individual users exit traffic flow (within a small interval) at each point in time is directly proportional to the speed of the flow in the network at that point in time, and inversely proportional to their trip length in the network. Fosgerau (2015) also proposes a bathtub model for the network based on the utility that users gain by departing the origin later and arriving to the destination earlier in time. The decreasing relationship between the speed and the density of vehicles in urban networks is then used to sort trips with identical timing according to the length of the trips in the network. A special case of the equilibrium problem with no late departures permitted is also analytically solved for Greendsheilds' relation in Arnott et al. (2016).

In this research, we extend Vickrey's congestion theory to model the dynamics of congestion in urban networks on a macro-level. For this purpose, the MFD model is employed to approximate the network outflow at each point in time as a function of the instantaneous vehicular accumulation of the system in the morning commute problem with a general distribution of schedule preferences over time for identical users. (1) The proposed approach improves the accuracy of the classic bathtub model by relaxing the simplifying assumption that allows approximating the travel time of the users just based on the traffic condition of the network on their arrivals to the region. The bathtub model presented in this