



Notes on the stability of dynamic economic systems

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Abstract

The Lyapunov function method is the focus of this paper in proving marginal stability, asymptotical or globally asymptotical stability of discrete dynamic systems. We show that the slightly relaxed versions of the well known sufficient conditions are also necessary. © 2000 Elsevier Science Inc. All rights reserved.

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1. Introduction

Stationary discrete dynamical systems can be mathematically formulated as

$$\underline{x}_{t+1} = \underline{g}(\underline{x}_t), \quad (1)$$

where \underline{x}_t is the state of the system at time period t , and \underline{g} is the state-transition function. If $S \subseteq \mathbb{R}^n$ is the state space, it is usually assumed that $\mathcal{D}(\underline{g}) = S$, $\mathcal{R}(\underline{g}) \subseteq S$, and \underline{g} is continuous. If $\underline{x}_0 \in S$ is an arbitrary initial state, then equality (1) uniquely determines the state trajectory, \underline{x}_t , $t \geq 0$. An equilibrium of the system is defined as a state $\bar{\underline{x}} \in S$ such that

$$\bar{\underline{x}} = \underline{g}(\bar{\underline{x}}). \quad (2)$$

Therefore the equilibrium-problem of system (1) is equivalent to the fixed point-problem of function \underline{g} , and any existence theorem of fixed-points of

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vector variable, vector valued functions can be used to establish the existence of equilibria of discrete dynamic systems.

In most applications the asymptotical behavior of the state x_t is investigated. These stability concepts are usually applied: marginal stability, asymptotical stability, and global asymptotical stability.

An equilibrium \bar{x} is called marginally stable if for arbitrary $\epsilon > 0$, there is a $\delta > 0$ such that $\|x_0 - \bar{x}\| < \delta$ implies that for all $t \geq 0$, $\|x_t - \bar{x}\| < \epsilon$. An equilibrium \bar{x} is called asymptotically stable if it is marginally stable and there exists a $\Delta > 0$ such that $\|x_0 - \bar{x}\| < \Delta$ implies that $x_t \rightarrow \bar{x}$ as $t \rightarrow \infty$. An equilibrium \bar{x} is called globally asymptotically stable if it is marginally stable and $x_t \rightarrow \bar{x}$ as $t \rightarrow \infty$ with arbitrary $x_0 \in S$.

Notice that asymptotical stability can be viewed as the local convergence of the iteration process generated by function \underline{g} , and similarly, global asymptotical stability can be interpreted as the global convergence of iteration sequences with the additional condition that the entire iteration sequence must remain close to \bar{x} .

There are many sufficient conditions that guarantee the marginal stability, asymptotical stability, or the global asymptotical stability of an equilibrium. Most of such conditions belong to one of the following classes: monotone iterations, conditions based on the Jacobian of \underline{g} , and the use of Lyapunov functions. Unfortunately, most conditions are only sufficient, and very few necessary stability conditions are known from the literature. Recently, Zhang and Zhang [1] have introduced a practical necessary condition based on the Jacobian of \underline{g} . However their result can be used only in very special cases. (See Ref. [2].)

In this paper we will focus on the Lyapunov function method and will introduce necessary stability conditions which are only slight modifications of the corresponding sufficient conditions. Based on the results of this paper the marginal stability, asymptotical stability, or the global asymptotical stability of an equilibrium can be analyzed for practical systems. For continuous systems, necessary conditions involving Lyapunov functions have been earlier presented in Ref. [3]. The results of this paper can be considered as the discrete time-scale counterparts of the classical theorems.

2. Sufficient conditions

Introduce the notation

$$\Omega = \{x \mid \|x - \bar{x}\| \leq \epsilon_0\} \quad (3)$$

with some $\epsilon_0 > 0$ and assume that $\Omega \subseteq S$. Let $V : \Omega \rightarrow \mathbb{R}$ be a real valued function defined on Ω . Introduce the following function properties:

- (a) V has a unique minimum at \bar{x} ;
- (b1) V is continuous at \bar{x} ;

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