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Self-organization in a model of economic system with scale invariant interactions

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Abstract

The method of constructing the local scale invariant stochastic models is proposed. The possible extension of minimal scale-invariant interaction principle for stochastic systems is formulated. A simple scale invariant model that possesses an economical interpretation is considered. Essential characteristics of its self-organization mechanisms are discussed.

1. Introduction

The gauge invariance is considered as one of the fundamental interaction principle in the theory of elementary particles [1]. There are reasons to suppose that it can be used for the construction of the models of basic self-organization mechanisms of complex adaptive natural system [2–5] and particularly for modeling of economical behavior [6].

We propose a simple way to construct models of such a kind using the principle of scale invariance in the stochastic dynamics. We suggest the use of ideas of minimal gauge interaction developed in the quantum field theory. It makes it possible to essentially reduce the number of arbitrary parameters so that the dynamical variables and parameters of the models allow simple and clear interpretation. The most complicated problem in the studies of complex system dynamics arises for unordered non-stable interaction structure. It is mostly the case in real economics. The activities of elements of economical system cannot be presented as being placed on the sites of a D-dimensional lattice and the basis for the solid state physics near neighbors

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interaction principle seems to be non-adequate in this situation. The interaction structure in economics is also usually time dependent on complex laws of dynamics.

The simple and often seemingly fruitful investigation approach for the system of such a kind is based on the idea to replace the interaction between elements with an interaction of them with an external stochastic communication field. The main problem is to define correct statistical properties of communication field, but in many cases the fundamental physical principles help to solve it. In our consideration, we use the following conception. The essential feature of the economical interactions is that they are realized on the basis of comparison of results of activities of the system elements. The convenient method of comparison is to use the scale. Thus, the scale becomes the transfer-agent for element interactions. The most universal economical scale is money, and from the physical point of view it is the "field" realizing interactions in economics.

We obtain the following picture. The interaction of elements in economical system is described with "scale field". It is natural to suppose that the local scale invariance of the system, means that the system behavior must be independent of the way in which it is described by a chosen scale field. It could be considered as a particular formulation for economical system modeling of general Einstein relativity principle. Our aim is to investigate the main features of the models with local scale invariance and to construct the most general local scale invariant stochastic model. It will be done in the next section. We show that the local scale invariance makes a rather strong restriction on the possible interactions structure and on the form of an observable in stochastic dynamical systems. In Section 3, we consider the simplest local scale invariant model and after that we discuss the possible ways of investigations in the framework of proposed approach.

2. Model

We consider the stochastic dynamical system of fields $\varphi_i(t)$, $i=1,\ldots,n$, evolving in time as follows:

$$\partial_t\varphi_i(t)+v_i(t)\varphi_i(t)+\sum_{j=1}^{n}\Lambda_{ij}(t)\varphi_j(t)=h_i(t),\quad i=1,\ldots,n\,. \tag{1}$$

where $h_i(t)$ are the Gaussian stochastic fields and all the diagonal elements of matrix $\Lambda(t)$ are zero: $\Lambda_{ii}(t)=0$. It is suppose to use the "material" fields $\varphi_i(t)$ in modeling for the presentation of results of economical activity. The field $h_i(t)$ presents the external stimulating activity force. The matrix element $\Lambda_{ij}(t)$ describes the direct influence of the jth element system on the ith one. The field $v_i(t)$ characterizes the time scale for the field $\varphi_i(t)$ dynamics.

If the initial condition for the fields $\varphi_i(t)$ is given thus:

$$\varphi_i(0)=c_i\,, \tag{2}$$