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A model for the evolution of economic systems in social networks

A.S. Elgazzar[a,b,*]

[a] *Max Planck Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, D-01187 Dresden, Germany*
[b] *Mathematics Department, Faculty of Education, 45111 El-Arish, Egypt*

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Abstract

A model for the evolution of economic systems is defined on a one-dimensional lattice using Pareto optimality. Pareto optimality is shown to maximize the total payoff of all agents in comparison to the Nash optimality. The small-world networks are found to be closer to the real social systems than both regular and random lattices. Then, the model is generalized to small-world networks that display different dynamics from the one-dimensional case. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The diffusion of a new concept (a technology, information, disease, etc.) through social and economic systems is a complex process that typically forms waves or avalanches. Also, it usually displays rich dynamics that is attracting the interest of many mathematicians and theoretical physicists [1]. Here our interest is restricted to the diffusion of a new technology through an economic system.

Generally the real economic system is a population of agents each of them has a technological level. Every agent is assumed to interact with a group of collaborators (neighbours) obtaining payoffs (profits). The base payoff is assumed to be higher for the higher technological levels, and the payoffs have to be bounded. Also, if an agent

* Corresponding author. Max Planck Institut fur Physik, komplexer Systeme, Nothnitzer Strasse 38, D-01187 Dresden, Germany. Tel.: +49-351-871-1211; fax: +49-351-871-1999.
E-mail address: elgazzar@mpipks-dresden.mpg.de (A.S. Elgazzar).

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is too advanced or too backward relative to his (her) neighbours, he (she) always has an incompatibility cost.

Every agent has to modify his (her) technological level to obtain the best payoff. After a long time of interaction, the population reaches a state at which each agent is satisfied with his (her) payoff. Then an improved technological level is introduced by an agent. Usually the new technology has a cost. The neighbours of this agent update their technological levels according to the new conditions. They have the possibility to accept or refuse the new technology. The new technology may diffuse through the whole population making a uniform front for cheap updating cost. On the other hand, if the cost is very high, only few agents can modify their levels. For intermediate cost values, the updating process continues through the population making an avalanche until another stable state is reached, and so on.

The avalanche's size, s is the number of agents that updated their levels in a step. It has been shown that [2] the avalanches size and several aspects of social and economic systems can be described in terms of power law distribution as,

$$P(s) \propto s^{-\gamma}, \quad \gamma \geqslant 1 \,. \tag{1}$$

Towards a theoretical approach to this phenomena, an important step was taken by Arenas et al. [3], where a general model for the evolution in socio-economic environments is introduced. Their model considers a population of agents distributed on a regular one-dimensional (1d) lattice with periodic boundary conditions. Only local interactions are considered. Initially each agent i is proposed to have a random positive real variable a_i which represents his (her) technological level. Each agent interacts with his (her) nearest neighbours, $j = i \pm 1$, obtaining payoffs calculated according to the following function:

$$\pi(a_i, a_j) = \begin{cases} a_i - k_1(1 - \exp-(a_i - a_j)) & \text{if } a_i \geqslant a_j \,, \\ a_i - k_2(1 - \exp-(a_j - a_i)) & \text{if } a_i < a_j \,, \end{cases} \tag{2}$$

where k_1 and k_2 represent the incompatibility costs resulting from being too advanced or too backward, respectively. The payoff of the agent i, π_i is

$$\pi_i = \pi(a_i, a_{i-1}) + \pi(a_i, a_{i+1}) \,.$$

At each time step, the technological level of a randomly chosen agent i is increased by a random variable $\Delta \in (0, 1)$. Each of the nearest neighbours, $j = i \pm 1$ has to update his (her) technological level. Agents are supposed to use Nash updating rule, which is defined as follows:

Definition 1 (Nash optimality)**.** If the payoff of an agent is not the greatest payoff with respect to his (her) nearest neighbours, he (she) changes his (her) technological level to the level of one of his (her) nearest neighbours that raises his (her) payoff.

This means the agent j selects the technological level which maximizes his (her) payoff from $\{a_{j-1}, a_j, a_{j+1}\}$.