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On the attractivity of a class of homogeneous dynamic economic systems☆

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Abstract

The attractivity properties of the set of equilibria of a special class of homogeneous dynamic economic systems are examined. The nonlinearity of the models and the presence of eigenvalues with zero real parts make the application of the classical theory impossible. Some principles of the modern theory of dynamical systems and invariant manifolds are applied, and the local attractivity of the set of equilibria is verified under mild conditions. As an application, special labor-managed oligopolies are investigated.

Keywords: Stability; Attractivity; Invariant manifold; Attractor; Labor-managed oligopoly

1. Introduction

Dynamic economic systems have been analyzed by many researchers during the last decade. Among the different model types the most attention has been given to dynamic oligopolies. Okuguchi [11] presented a comprehensive summary of single-product oligopolies without and with product differentiation and also gave a detailed analysis of earlier works on the subject. The existence and uniqueness of the equilibrium is examined and the stability of the equilibrium is analyzed with discrete and continuous

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time scales. The extensions of the models and results for multi-product oligopolies were presented in Okuguchi and Szidarovszky [12], where the different variants of the Cournot model are also discussed including labor-managed oligopolies, rent-seeking games, and models with production adjustment costs.

In this paper the asymptotic behavior of a special labor-managed oligopoly will be examined. The problem can be formulated as follows.

Let us consider an n-firm industry, where all firms are labor-managed. Let us assume the hyperbolic price function

$$p(s) = \frac{b}{s},$$

where s is the total output of the industry, and linear production functions l_i, and linear labor-independent cost functions c_i:

$$l_i(x_i) = a_i x_i \quad \text{and} \quad c_i(x_i) = \alpha_i x_i + \beta_i,$$

where x_i is the output of firm i $(i = 1, \ldots, n)$.

Economic interpretation requires that all parameters b, a_i, α_i, and β_i be positive. The surplus per unit of labor for firm i is given by

$$\begin{aligned}\phi_i(x_1, \ldots, x_n) &= \frac{x_i p(s) - w l_i(x_i) - c_i(x_i)}{l_i(x_i)} \\ &= \frac{b}{a_i(x_i + Q_i)} - w - \frac{\alpha_i}{a_i} - \frac{\beta_i}{a_i x_i}, \end{aligned} \tag{1.1}$$

where $Q_i = \sum_{l \neq i} x_l$ is the output of the rest of the industry, and w is the competitive wage rate. This economic situation can be modeled as an n-person game where the set of strategies for each firm is the interval $X_i = [0, \infty)$ and the payoff function of firm i is ϕ_i.

The existence of positive equilibria will be first examined and their asymptotic behavior will then be analyzed. We will show that there are infinitely many positive non-isolated equilibria under appropriate conditions. As the equilibrium set is connected, the classical Lyapunov theory cannot be used to analyze the asymptotic behavior of the equilibria. It will turn out that the modern theory of dynamical systems and invariant manifolds serves as a useful technique in our case.

This paper is developed as follows. In Section 2 we will examine the existence of equilibria and give a complete description of the equilibrium set. Then the dynamic extension of the model will be introduced with continuous time scales. The major attractivity properties of the equilibria will be formulated in Section 3. In Section 4 we will introduce and discuss the main theoretical issues and then apply these results in Section 5 to analyze the asymptotic behavior of the equilibrium set in a special class of homogeneous systems that includes our dynamic model as a special case. In Section 6 we will present an elementary proof of the strong attractivity part of our main result based on simple techniques in solving homogeneous systems.