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A stochastic differential game of capitalism

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ABSTRACT

We develop a stochastic differential game of capitalism to analyze the role of uncertainty. In the deterministic game, the firm's rent is completely taxed away and the firm stops investing completely. In the stochastic game, the government does not tax the firm's rent completely. The firm posts a positive rate of investment if the firm's rent exceeds the labor's income. Although the cooperative solution is indeterminate, cooperation is always Pareto optimal compared to the non-cooperative Markovian Nash equilibrium. For individual rationality, we apply a payoff distribution procedure based on Yeung and Petrosyan (2006) to derive a subgame-consistent solution.

1. Introduction

The recent global financial crisis has brought back the debates on the merits of capitalism, especially the uncertainties associated with a capitalist economy. However, current differential games of capitalism are all deterministic. Since the future is inherently unknown, it is inevitable that the decisions of the individual players over an infinite time horizon may be affected by uncertainties about the future, such as those brought about by a financial crisis. In addition, the capital accumulation process in capitalism may be subjected to uncertainties such as environmental disasters and the depletion of resources. In this paper, we explore the consequences of introducing uncertainty into a differential game of capitalism.

Differential games of capitalism were first explored by Lancaster (1973) who adopted a two player non-cooperative differential game where the workers control the share of their consumption in total output while the capitalists control the share of investment in the surplus. Comparing the Nash equilibrium with the cooperative solution (from maximizing a weighted sum of worker and capitalist consumption), Lancaster demonstrated that both players obtain more consumption under cooperation, hence resulting in the dynamic inefficiency of capitalism. Lancaster's work has been extended by others. Shimomura (1991) characterized the feedback equilibrium of the game. Kaitala and Pohjola (1990) considered a variation on the original Lancaster model in which the politically powerful group of workers controls redistribution while the economically powerful group of capitalists controls accumulation. Grim trigger strategies are employed by both groups to sustain cooperation as an equilibrium. In their model, the workers and capitalists receive returns equivalent to the labor and capital share, respectively. Dockner et al. (2000) surveyed similar models and extensions. The model in the current paper is a variation on the game of capitalism in the sense that the government represents the workers while the firm represents

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the capitalist. As such, our approach is similar to that of Kaitala and Pohjola (1990). However, two features in the current specification can be distinguished. Firstly, the government in this present model is a vote-maximizer. This is a significant departure from conventional economic models, in which the government is typically characterized as a benevolent dictator with the aim of maximizing social welfare. The government in this model deals with the long-run political "tradeoff" between economic growth and distributional equity. Vote-maximization is not to be taken literally to imply a democracy. Instead, the vote function in this chapter can be interpreted as a function for political support. Secondly, the firm draws its capital from its shareholders and must earn an after-tax return to be distributed to these shareholders as returns for their investment. Such a characterization of the firm is arguably closer in spirit to capitalism than those in earlier games of capitalism.

A key conclusion in differential games of capitalism is: capitalism is dynamically inefficient and the cooperative solution is superior. In this paper, we consider whether such a conclusion still holds in the presence of uncertainty. Specifically, we develop a stochastic differential game of capitalism by applying methods in stochastic calculus, an area that has been predominantly studied in finance and mostly applied in stochastic control problems (Itô, 1951; Oksendal, 2003; Chang, 2004). We demonstrate that many conclusions in the deterministic setting can be undermined if stochastic elements are incorporated into the model. We draw on the stochastic Solow equation developed by Merton (1975) and elaborated in Chang and Malliaris (1987). Merton (1975) considered a one-sector neoclassical growth model of the Solow-type where the dynamics of the capital–labor ratio can be described by a diffusion-type stochastic process. The particular source of uncertainty chosen is the population size. Using the Reflection Principle, Chang and Malliaris (1987) demonstrated the existence and uniqueness of the solution to the classic Solow equation under continuous time uncertainty for the class of strictly concave production functions which are continuously differentiable on the non-negative real numbers. This class contains all CES functions with elasticity of substitution less than unity. A steady state distribution also exists for this class of production functions with a bounded slope at the origin. A condition on the drift–variance ratio of the stochastic differential equation alone, independent of technology and the savings ratio, is found to be necessary for the existence of a steady state. In contrast to both Merton (1975) and Chang and Malliaris (1987), we analyze a stochastic differential game in continuous time and the chosen source of uncertainty arises from capital accumulation.

Under the stochastic Markovian Nash equilibrium, the government will tax less than the full amount of the rent accrued to the firm, which will post a positive rate of investment. It is not only necessary that the investment rate is positive. The firm's rent must also exceed the labor's income. The stochastic Markovian Nash equilibrium is significantly different from the Markovian Nash equilibrium obtained under deterministic conditions. In the deterministic case, the rent of the firm is completely taxed away and the firm stops investing completely, which is a very extreme and unrealistic solution. Introducing uncertainty into the model thus produces a solution that is less extreme and hence more realistic.

Cooperation in this paper is not collectivism in the Marxist or communist sense as is the case in earlier papers. Instead, cooperation in this context would be some arrangement in which the profit-maximizing firm would be willing to work with the vote-maximizing government to achieve a Pareto optimal solution. Although it is not possible to determine the cooperative solution from the model, we are able to prove that the cooperative solution is always Pareto optimal compared to the Markovian feedback Nash equilibrium. Furthermore, we apply a payoff distribution procedure by Yeung and Petrosyan (2006) to derive a subgame-consistent solution in the stochastic game of capitalism and ensure individual rationality.

The rest of this paper is organized as follows. In Section 2, we derive the stochastic capital accumulation equation and formulate the stochastic game of capitalism. The deterministic and stochastic Markovian Nash equilibrium are derived in Section 3. Section 4 discusses whether stochastic cooperative solution is preferred to the Markovian Nash equilibrium. Finally, Section 5 presents the concluding remarks.

2. The stochastic differential game of capitalism

Consider a differential game of capitalism with two players, a government and a representative firm.

2.1. Structure of the economy

The economy has a neoclassical production function, which is represented in intensive form as $y = f(k), f'(k) > 0, f''(k) < 0$, $\lim_{k\to 0}[f'(k)] = \infty$, $\lim_{k\to\infty}[f'(k)] = 0$. The labor force receives an income equal to its marginal product $f(k) - kf'(k)$ while the firm derives a rent equivalent to its marginal product $f'(k)$.

2.2. The government and the firm

The government is a vote-maximizer: it will adopt policies that will best assure its continuation in power, increase its political support or improve its vote-getting power. This is represented using a vote function $v[k, x(\cdot), s(\cdot)]$, where $x(\cdot)$ represents the tax or social transfer within the government's control while $s(\cdot)$ represents the investment rate controlled by a representative firm.

The government's objective functional can thus be expressed as follows:

$$\max_{x(.)} J^G(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} v[k, x(\cdot), s(\cdot)]\,\mathrm{d}t.$$