Decision Support

An analysis of insurance demand in the newsboy problem

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\textbf{A B S T R A C T}

In this paper we study the standard newsboy problem, but under two new assumptions when compared to the existing literature. First, we assume that the wholesaler is an expected profit maximiser who sets the wholesale price optimally, and in doing so, takes into account the salvage value at which the newsboy can return unsold items to the wholesaler. Second, we assume that the salvage value is a choice variable of the newsboy, and in that way, it acts as a standard insurance device. The newsboy's optimal salvage value then represents an optimal demand for insurance. We study in particular the optimal pricing problem of the wholesaler, and show that it can be expressed as a mark-up equation. We also show that insurance is provided at an actuarily unfair price. As regards the optimal demand for insurance by the newsboy, the problem is too complex for a closed form solution to be possible, so we resort to a simulation which returns the results that a strictly positive level of strictly partial insurance is demanded when the newsboy is strictly risk averse, and the optimal level of insurance coverage increases with risk aversion.

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\textbf{1. Introduction}

In this paper we consider the classic newsboy inventory problem, with a weakly risk averse newsboy.\textsuperscript{1} The introduction of risk aversion to the problem is, of course, not new, with the seminal paper having been published more than 40 years ago (Baron, 1973), and the most complete analysis having been published 20 years ago (Eeckhoudt, Gollier, & Schlesinger, 1995). Several other papers that include a risk averse newsboy have appeared since then (e.g. Agrawal & Seshadri, 2000; Wang, Webster, & Suresh, 2009). A detailed survey can be found in Wang, Webster, and Zhang (2012). There is, however, one aspect of the risk averse newsboy problem that does not (to the best of our knowledge) appear to have been studied, and that is the question of insurance. Of course, there are plenty of papers that have looked at risk-sharing between the newsboy and the supplier (e.g. Cachon & Lariviere, 2005; Gan, Sethi, & Yan, 2004; Tsay, 2002). But no paper thus far explicitly considers the relationship between the newsboy and the supplier in terms of insurance. Notwithstanding the lack of attention to insurance, a clear insurance mechanism has been a standard inclusion in many papers, namely a salvage value for unsold units. In the present paper we focus on using the salvage value as an insurance device, and we look at the optimal insurance contracts using that device.

As an integral part of the paper, we also allow not only the newsboy to make optimal choices, but also the wholesaler (the supplier to the newsboy, who is the retailer). Specifically, we assume that the wholesaler sets the wholesale price of units sold to the newsboy optimally, with the objective of maximizing expected profit given the newsboy's choices. This is also an aspect of the newsboy problem that has not been included in many previous papers, where the wholesaler exists only as an exogenous parameter set, normally only the wholesale price that the newsboy purchases at. Notable exceptions are Pasternack (1985) for the case of a risk neutral newsboy, and with risk aversion, Lau and Lau (1999) and Lariviere and Porteus (2001). However, these papers are mainly concerned with supply chain coordination rather than supplier profit maximisation.\textsuperscript{2} It is, however, an integral part of the present paper, as it allows us to study a proper market environment in which insurance, as represented by the salvage value for unsold items, becomes a choice variable for the newsboy. Other

\textsuperscript{1} The authors are grateful to four anonymous referees whose comments and suggestions have served to eliminate several errors and in general have substantially improved the paper. Any remaining errors are the sole responsibility of the authors.

\textsuperscript{2} In unpublished work, Tang and Rudi (2010) have a very interesting paper that does look at optimal wholesaler pricing under the objective of maximisation of expected profit.

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papers have made the salvage mechanism a choice variable for the wholesaler; see, for example, Pasternack (1985), and Lau and Lau (1999). We discuss this option below, but we note that since our objective is to consider insurance, we are more interested in allowing the insurance consumer (the newsboy) choose his optimal insurance demand, given the conditions set by the insurance provider (here, the wholesaler). Cachon and Kök (2007) provide a model in which the newsboy does implicitly choose the insurance conditions, but instead of being insured by the wholesaler, the insurance happens directly in the market for the good, by allowing items unsold in the first instance to be offered at a discount later on. We do not allow the wholesaler to charge separately for the insurance mechanism that is represented by the salvage value re-purchase system, although we are very interested in how the insurance mechanism affects the optimal wholesale price for units of product sold to the newsboy.

In the paper we consider the comparative statics of the optimal level of insurance, r∗, as chosen by the newsboy. In particular, we are interested in how insurance demand changes with the newsboy’s risk aversion. Notice that, in principle, the effect is ambiguous, since not only does risk aversion affect the optimal desire for insurance, it will also affect the optimal per-unit price that the wholesaler charges.

While we use the newsboy problem to study both optimal wholesaler choices along with optimal retailer (in our case, the newsboy) choices when insurance is an integral aspect, there are other scenarios that also adhere to the same general scheme. Another clear example of the same is when a retailer sells a good along with a guarantee of some sort, but where the guarantee is not charged for separately from the actual good.

2. Assumptions

Following closely the existing literature, the newsboy is a retailer who sells a perishable good for which the consumer demand is risky. The newsboy orders in product from a supplier, who we refer to as the wholesaler, and of essence is the assumption that the order must be placed (and paid for) before consumer demand is known. Thus the newsboy’s optimal order needs to balance the risk of overstocking (when the amount of product purchased from the wholesaler exceeds demand) against the risk of understocking (when there is unfilled demand at the amount of product purchased from the wholesaler). The wholesaler charges a wholesale price of c for each unit of product sold to the newsboy, and offers to buy back unsold units at a salvage value of r dollars per unit. In this way, we have an insurance mechanism built into the model for the overstocking risk, but the newsboy is not offered any external risk mitigation device for the understocking risk. Indeed, in the model of this paper there is no financial cost for lost sales when understocking occurs. A lost sale is valued at 0. This assumption, which while it certainly implies a minor restriction of generality on the analysis, allows us to focus our attention on the insurability of the overstocking risk, which without doubt is the risk that is most realistically insurable as it involves a clear financial cost.

The newsboy sells units to the market (i.e. to consumers) at an exogenously determined retail price p. We restrict 0 ≤ r ≤ c ≤ p. For reasons set out below, we also assume r < p. We assume that the wholesaler’s cost function is restricted to a constant marginal cost of q per unit supplied to the newsboy, with no fixed cost.

Consumer demand is given by x, which is a random variable distributed according to pdf f(x) and distribution F(x). This density and distribution would be changed by a change in the retail price p, but we will hold p constant in all that we do, and so we avoid making arbitrary assumptions about f as a function of p. Essentially, we simply take the retail price as given (which of course is not a bad assumption for the particular case of newspapers, which normally are sold at a given price each day), and take the probability density function to be that corresponding to the given retail price. We assume bounded demand, and we normalise our units so that demand is defined on the unit interval, i.e. 0 ≤ x ≤ 1, that is, demand is normalised with respect to the maximal possible amount. Of course, the newsboy must choose his order quantity, y, before the consumer demand is known. Clearly, the normalisation of x also implies bounds on the order; 0 ≤ y ≤ 1. The final sales made to consumers is s = min(x, y).

We interpret r = 0 as no insurance, r = c as full insurance, and 0 < r < c as partial insurance. Of course other types of insurance could easily be modelled. For example, the newsboy might be given a choice of how many units of the order are able to be resold back to the wholesaler at a set price of r. But in the present paper we concentrate only upon the simpler case of all unsold units being refunded, which appears to be the more realistic scenario. We firstly study a base-line model in which r is an exogenous parameter, in order to see how r affects the optimal choices of c (by the wholesaler) and of y (by the newsboy), and then we tackle the problem of allowing the newsboy to set both y and r.

The model works as follows. The wholesaler sets the wholesale price, c, under full information on the newsboy’s utility and the other parameters in the model. If we denote by c∗ the wholesaler’s optimal price, then we can write c∗(y, r) to capture the fact that the optimal wholesale price is calculated as a function of the newsboy’s order, y, and the salvage value price, r. The newsboy then takes c∗(y, r) and makes an optimal choice of either just the order quantity, y (in the baseline model), or both the order quantity and the salvage value price, r (in the insurance demand model), thereby closing the model off with optimal values of y∗, r∗ (if r is a choice variable), and consequentially c∗(y∗, r∗).

Since we are interested in insurance, we assume that the newsboy is not risk loving – he is either risk averse or risk neutral, or more succinctly, the newsboy is weakly risk averse. The newsboy’s utility function for wealth w is u(w), where we assume u′(w) > 0 and u′′(w) ≤ 0. The newsboy is assumed to maximise expected utility. On the other hand, we assume that the wholesaler is risk neutral, and maximises her expected monetary profit.

3. The newsboy’s problem

While now well-known in the literature, we firstly review the expected utility maximisation problem of the newsboy under the assumption that r is not a choice variable. Given c and r, the newsboy’s expected utility of an order of size y is

\[ E[u] = \int_0^y u(x(p-c) + (y-x)(r-c)) f(x)dx + \int_y^\infty u(y(p-c)) f(x)dx \]

\[ = \int_0^y u(x(p-c) + (y-x)(r-c)) f(x)dx + u(y(p-c))[1-F(y)]. \]

These assumptions, for example, follow closely the baseline model in Eeckhoudt et al. (1993). However, Eeckhoudt et al. (1995) do allow the newsboy to re-stock at a higher price after demand is known. Nevertheless, even under that extension no specific cost is associated with understocking of units. Other papers do include an embedded cost component (a penalty cost) to account for lost profits when demand exceeds the order quantity. See, for example, Khoury (1999), Section 2.

Again, the assumption that the demand distribution is bounded aids the tractability and clarity of the model, without any notable loss in generality, at least as far as the issue of existence of optimia and their comparative statics effects that we are interested in here are concerned. Bounded demand is also a relatively common assumption in much of the existing literature (see, for example, Dionne & Mounis, 1996; Eeckhoudt et al., 1995).

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