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# A new method to control chaos in an economic system

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ABSTRACT

In this paper, the method to control chaos by using phase space compression is applied to economic systems. Because of economic significance of state variable in economic dynamical systems, the values of state variables are positive due to capacity constraints and financial constraints, we can control chaos by adding upper bound or lower bound to state variables in economic dynamical systems, which is different from the chaos stabilization in engineering or physics systems. The knowledge about system dynamics and the exact variety of parameters are not needed in the application of this control method, so it is very convenient to apply this method. Two kinds of chaos in the dynamic duopoly output systems are stabilized in a neighborhood of an unstable fixed point by using the chaos controlling method. The results show that performance of the system is improved by controlling chaos. In practice, owing to capacity constraints, financial constraints and cautious responses to uncertainty in the world, the firm often restrains the output, advertisement expenses, research cost etc. to confine the range of these variables' fluctuation. This shows that the decision maker uses this method unconsciously in practice.



## 1. Introduction

Since Ott et al. [1] introduced the OGY control method, researchers are increasingly interested in controlling chaos of the nonlinear systems. In recent years, a lot of studies have been devoted to this topic, such as Boccaletti and Grebogi [2], He and Westerhoff [3], Holyst and Urbanowicz [4], Li et al. [5], Kopel [6], Liu and Wang [7], Sheng et al. [8], Song et al. [9], Stoop and Wagner [10], Wagner and Stoop [11], Wieland and Westerhoff [12], Xiang et al. [13] and Zhang and Shen [14,15]. As we know, economic systems are nonlinear and may display chaotic behavior (see [12,16–30]). When some systems' dynamics are chaotic, some players' performance decreases contrasted with the equilibrium. Generally, these players should adapt some certain methods to control the chaos. At present, most of methods for chaos control are designed for the physics systems, which are mainly used in the natural science and engineering. For the chaos control in the economic systems, there only are a few introductions. Ahmed et al. [31] and Agiza [32] use OGY method to control the chaos in economic systems. Kass [33] associates the chaotic targeting method with the OGY method to stabilize chaos in a dynamical macroeconomic model. Kopel [6] uses the chaotic targeting method to control chaos of a monopoly output adjustment model. Holyst and Urbanowicz [4] uses delayed feedback control method (DFC) to control chaos in a duopoly investment model. Wieland and Westerhoff [12] apply the OGY method and DFC separately to stabilizing chaos in an exchange rate dynamic model. In fact, through the control parameter's perturbation, the original OGY method forces the unstable orbits to become stable on the stable manifolds of hyperbolic fixed points. It requires the unstable fixed points are dependant to the control

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parameters. Furthermore, the stable manifolds lie in the points. While such information may be identified from observations in natural science applications, chaos control in an economic context is often seen as rather critical. The chaotic targeting method also requires knowledge of the map and its fixed point, so this method is not convenient in its practical using. DFC avoids fancy data processing used in the OGY method, and its application is very straightforward. Moreover, DFC represents a self-adaptive behavior of economic entities. DFC may be the only chaos control method satisfying with the basic economic features except the limiters methods introduced by Wieland and Westerhoff [12].

The limiter method is explored by Wagner and Stoop [11], Zhang and Shen [14,15], Stoop and Wagner [10]. Zhang and Shen [14,15] call it as phase space compression. One advantage of the limiter method is that it does not add complexity to the system by increasing the size of the system's state space [3]. Another advantage is that stabilization may be achieved by infrequent interventions. The limiter method has realized the control of chaos and hyper-chaos through limiting of the strange attractor's space in chaos and hyper-chaos system. Actually, this is to restrain the system variable values in a subset. In most economic systems, some state variables can be controlled in certain area by the object, such as the output, the research investment, the advertisement cost, price and so on. All these can help us to control chaos in some economic systems using the limiter method.

He and Westerhoff [3] studied chaos control of economic systems using limiter method. They discussed commodity markets by using price limiters. They have achieved chaos control through giving price upper limiter or lower limiter. It is worthy to be noticed that the economic model studied by He and Westerhoff [3] is one-dimension. In two-dimensions, especially in imperfect competitive market, the problems need to be studied furthermore, such as whether the players should carry out the control and whether it needs all players to take control actions in order to control chaos successfully.

The remainder of this paper is organized as follows. Section 2 presents the limiter method in multi-dimensional economic systems. In Section 3, we introduce a dynamic output game with bounded rationality and examine the dynamics of the model by using stability and bifurcation analysis. Section 3 also gives performance indices measuring the performance of economic systems and shows the results from numerical simulations. In Section 4, we use the control method to control the chaos of output model and give the comparison for the players' income before the control and the income after the control. The final section concludes the paper.

## 2. The limiter method in economic systems

Wagner and Stoop [11], Zhang and Shen [14,15], and Stoop and Wagner [10] put forward the limiter method (phase space compression) to control chaos in physics systems. We will introduce this method's applications in certain economic systems.

Consider the following discrete-time dynamic system:

$$X_i(t+1) = M_i(X_1(t), \ldots, X_N(t)), \quad i = 1, \ldots, N. \tag{1}$$

In (1), $t = 1,2,\ldots$, is the discrete-time variable. $X_i$ is a state variable, and $M_i$ is a mapping of $R^N \mapsto R$, $i = 1,2,\ldots,N$. Let the chaotic solution be in certain space for system (1), the trajectory of the solution is expressed by strange attractors in phase space. From the characteristics of strange attractors, we know that the trajectory of the solution in system (1) will be limited in certain space $V$ with boundary. Then we can choose a nonempty subset $W$ of $V$, and $W \subset V$, and the solution of system (1) will be limited in space $W$. That is, the orbit evolvement can be controlled according to the following equation

$$X_i(t+1) = \min\{X_i^1, \max\{X_i^0, M_i(X_1(t), \ldots, X_N(t))\}\}, \quad i = 1,2,\ldots,N. \tag{2}$$

In (2), $(X_1^0, X_2^0, \ldots, X_N^0), \ (X_1^1, X_2^1, \ldots, X_N^1) \in W_o$.

As we know, the positive Lyapunov exponent $\lambda$ value is the essential characteristics of chaos or hyper-chaos in the nonlinear system. The positive $\lambda$ value reflects the divergence or expansion of nonlinear system's orbits on certain direction or torus. Through compressing the system orbits space and limiting the orbit's free divergence or expansion, the above method can realize the control of chaos and hyper-chaos state if positive $\lambda$ becomes negative in the controlling course.

If we use the above method in certain economic systems, we define an upper limit $X_i^1$ and lower limit $X_i^0$ for the value of variables. We compare the variable's values generated by the decision-making rule in (1) at time $t+1$ with the upper limit and lower limit. If the value is between the upper value and lower value, this value is the state variable's value at time $t+1$. If it is smaller than the lower value, the state variable's value is the lower limit; if it is bigger than the upper limit, state variable's is the upper limit at time $t+1$. Because this method is to control chaos through the restrict values with the upper limit and lower limit, we call it upper and lower limiter method.

In some economic systems, the value of the economic variables will not approach to infinite because of finite economic resources. In addition, there are many economic variables are non-negative, such as interest rate, output, advertisement cost, price, research investment and sales income. Therefore, the value of variables will not approach to negative infinite. That is to say, we can control chaos only by adding the upper limit and lower limit of state variables. We call the method of adding the lower limit to economic variables lower limiter method, while we call the method of adding the upper limit to economic variables upper limiter method.

The lower limiter method controls orbits of system according to the following equation:

$$X_i(t+1) = \max\{X_i^0, M_i(X_1(t), \ldots, X_N(t))\}, \quad i = 1,2,\ldots,N. \tag{3}$$