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Economic equilibrium problems in reflexive Banach spaces

Zdzisław Naniewicz*

Cardinal Stefan Wyszyński University, Faculty of Mathematics and Science, Dewajtis 5, 01-815 Warsaw, Poland

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Abstract

The problem is to minimize a finite collection of objective functions over admissible sets depending on the so-called price vector. The minima in question and the price vector are linked together by a subdifferential governing law. The problem stated as a system of variational–hemivariational inequalities, defined on a nonconvex feasible set, is reduced to one variational–hemivariational inequality involving nonmonotone multivalued mapping. The existence of solutions is examined under the assumption that the constrained functions are positive homogeneous of degree one. The study has been inspired by economic issues and leads to new results concerning the existence of competitive equilibria.

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1. Introduction

Let us consider a competitive market economy consisting of m consumers (indexed by $j \in J = \{1, \ldots, m\}$), n firms (indexed by $i \in I := \{1, \ldots, n\}$), and s goods (indexed by $l \in L := \{1, \ldots, s\}$). In such an economy, society's initial endowments and technological possibilities (i.e., the firms) are owned by consumers. The initial endowment of j's consumer is given by $\omega_j \in \mathbb{R}^s_+$. In addition, we suppose that consumer j owns a share κ_{ji} of firm i, where $\sum_{j \in J} \kappa_{ji} = 1$. Denote by $Y_i \subset \mathbb{R}^s$ the production set associated with i's firm.

* Tel.: +48 22 44 69 426; fax: +48 22 56 18 929.
E-mail address: naniewicz.z@acn.waw.pl.

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Recall that the allocation $(x_1^\star, \ldots, x_m^\star, y_1^\star, \ldots, y_n^\star)$, $x_j^\star \in \mathbb{R}_+^s$, $j \in J$, $y_i^\star \in \mathbb{R}^s$, $i \in I$ and price vector $p^\star \in \mathbb{R}_+^s$ constitute a competitive (or Walrasian) equilibrium if the following conditions are satisfied ([19]):

Profit maximization: For each firm $i \in I$, $y_i^\star$ solves

$$\max_{y_i \in Y_i} \langle p^\star, y_i \rangle; \tag{1}$$

Dis-utility minimization: For each consumer $j \in J$, $x_j^\star$ solves

$$\min \left\{ V_j(x_j)\colon \langle p^\star, x_j \rangle \le \langle p^\star, \omega_j \rangle + \sum_I \kappa_{ij} \langle p^\star, y_i^\star \rangle, x_j \in \mathbb{R}_+^S \right\}; \tag{2}$$

Market cleaning:

$$\sum_{j \in J} x_j^\star = \sum_{j \in J} \omega_j + \sum_{i \in I} y_i^\star. \tag{3}$$

Now observe that the market cleaning condition in the case of positive prices $p^\star > 0$ ($p_l^\star > 0$ for each $l \in L$) can be written as

$$\left.\begin{aligned} &\sum_{j \in J} x_{lj}^\star - \sum_{j \in J} \omega_{lj} - \sum_{i \in I} y_{li}^\star \le 0 \\ &\left(\sum_{j \in J} x_{lj}^\star - \sum_{j \in J} \omega_{lj} - \sum_{i \in I} y_{li}^\star \right) p_l^\star = 0 \end{aligned} \right\} \quad l \in L. \tag{4}$$

If we set $\phi_j(p) := \langle p, \omega_j \rangle + \sum_{i \in I} \kappa_{ij} \sup_{y_i \in Y_i} \langle p, y_i \rangle$ and

$$\Phi(p) := \sum_{j \in J} \phi_j(p) = \left\langle p, \sum_{j \in J} \omega_j \right\rangle + \sum_{i \in I} \sup_{y_i \in Y_i} \langle p, y_i \rangle, \quad p \in \mathbb{R}_+^s,$$

then (4) can be expressed in the form of variational inequality

$$\left\langle p - p^\star, -\sum_{j \in J} x_j^\star \right\rangle + \Phi(p) - \Phi(p^\star) \ge 0, \quad \forall\, p \in \mathbb{R}_+^s, \tag{5}$$

where Φ and ϕ_j are convex, nonnegative valued, positive homogeneous of degree 1 functions.

This observation has inspired the study of more general problem in which the market cleaning condition (3) is no longer required and is replaced by the variational inequality (5). It has an important meaning: the market clears for a commodity if its equilibrium price is positive. Otherwise, there may be an excess supply of the commodity in equilibrium and then its price will be zero.

Thus a more general problem can be stated:

Find $\{x_j^\star\}_{j \in J} \subset \mathbb{R}_+^{sm}$ and $p^\star \in \mathbb{R}_+^s$ such that

Dis-utility minimization: For each consumer $j \in J$, $x_j^\star$ solves

$$\min \left\{ V_j(x_j)\colon \langle p^\star, x_j \rangle \le \phi_j(p^\star), x_j \in \mathbb{R}_+^s \right\}; \tag{6}$$